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Span-wise and Sectional Flow Structure in Separation Bubbles Shown by Dynamic Mode Decomposition

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Abstract

Two-dimensional (2-D) flow structures both inside and outside of a separation bubble near the leading edge of a 2-D flat rectangular section were visualized. The flow was captured by an image within the plane normal (xz plane) and parallel to the side surface (xy plane). The image was converted to the timedependent velocity vector field using the PIV technique. Timeseries data were analyzed by dynamic mode decomposition (DMD). Vortex-dominant modes with convection and pulsating modes in the entire separation bubble were captured in the xzplane, and an undulation mode along the span-wise direction with convection was observed in the xy plane in turbulent flow. This implies significant momentum transfer in the lateral direction, which contributed to an increase in the span-wise correction of the flow structure.

Introduction

Separation bubbles are regions surrounded by shear flow, separating from the leading edge to the reattachment point downstream and the body side surface from the viewpoint of the time-averaged flow pattern. They have been investigated in numerous studies [3][5][7][8]: Kiya and Sasaki [6] reported the periodic fluctuation of a separation bubble and calculated its characteristic frequency in terms of the time-averaged reattachment length. They also clarified the span-wise correlated structure of flow velocity near the separation bubble. Jankauskas and Sankaran [4] reported the contribution of the separation bubble to the increase in the span-wise cross-correlation of pressure on the body side surface. Saathoff and Melbourne [9] focused on the separating shear layer near the leading edge followed by large-scale vortex flow, which caused an increase in the span-wise correlation. Shirato et al. [12] explored the crosscorrelation of the surface pressure for various two-dimensional (2-D) rectangular cross-section models, and proposed a relationship between the span-wise correction length of the buffeting force on a strip and that of the oncoming vertical wind velocity component. Based on wind tunnel experiments and CFD, Ito et al. [2] reported the contribution of vortex formation in wake to the correlation increase and proposed a universal crosscorrelation of the strip buffeting force, which can be applied to bridge cross-sections.

Previous studies, such as those above, indicated an increase in the span-wise correlation of surface pressure and buffeting force and proposed its mechanism. Development of the visualization technique enabled investigation of complex 2-D and 3-D flow structures. In this study, 2-D flow structures near and inside the separation bubble were visualized, and the time series of the velocity vector fields obtained using the PIV technique were analyzed using dynamic mode decomposition (DMD) [11]. Various modes were obtained, which enhanced the span-wise

(lateral) direction of the velocity component, and we discuss the mechanisms of the increase in the span-wise correlation.

Flow Visualization

All experiments in this study were performed in the wind tunnel of the Department of Civil and Earth Resources Engineering, Kyoto University. The working section of the wind tunnel measured 1.0 wide \times 1.8 m high. A 2-D rectangular cross-section model with a cross-section of B/D = 8 (B: width, D: depth of cross section, D = 0.038 m), and span length of 0.90 m was used to visualize the flow around the model under uniform flow (I_u <0.5%, I_u : longitudinal turbulence intensity) and grid turbulence ($I_u = 9.6\%$, $L_u = 0.1$ m, L_u : longitudinal scale of turbulence) conditions. The model was secured horizontally in the wind tunnel. Hereafter, the coordinates located at the leading edge in the mid-span of the model are denoted by x and u for the longitudinal, y and v for the lateral (span-wise), and z and w for the vertical coordinates and wind velocity components, respectively (see Figure 1).



Figure 1. Coordinate system and wind velocity components.

A slit 1 mm in width was made along the span-wise direction, 2 mm downstream from one of the leading edges, to emit smoke into the separation bubble. Smoke was supplied from the generator (8384; KANOMAX) to both ends of the model (see Figure 2). To visualize the approaching flow and the flow passing the separation bubble, smoke from the same generator was also supplied to a wing-shaped emitter 0.9 m upstream of the model. A laser sheet (PIV Laser 2,000 m/G-KD; Katou Koken) was positioned to visualize the flow within the *xy* and *xz* planes. The sheet in the xy plane captured the u and v components of the flow on the inside of the separation bubble over the ranges 0 < x/D <2.88 and 0 < y/D < 5.0. The distance from the side surface of the model to the xy plane was set at 2–6 mm. The sheet in the xzplane visualized the flow containing the separation bubble over the ranges -1.6 < x/D < 4.8 and 0 < z/D < 0.8. The flow pattern was recorded using the PIV camera (1024PCI Model 1K; Photron Fastcam) as snapshots with an interval of 0.002 s for 10-12.6 s. An image correlation method was used for the PIV analysis to

obtain the time series data of the velocity vector field. The mesh resolution was 0.04*D* in the *xy* plane, and 0.032*D* in the *xz* plane. The Reynolds numbers with respect to *D* in the flow visualization were 2.5×10^3 and 3.8×10^3 .



Figure 2. Smoke supply and slit to emit smoke to the separation bubble.

DMD

DMD [11] is able to decompose flow fields into modes showing the temporal and spatial evolution of flow patterns. Each mode has specific frequency and amplification characteristics (similar to damping). DMD modes also contain phase information between elements at different locations in DMD modes. DMD assumes that the time sequence of the physical quantity, { $v_t(\mathbf{x})$ } to { $v_{t+dt}(\mathbf{x})$ }, can be expressed by a linear mapping:

$$[A] \{ v_t(\mathbf{x}) \} = \{ v_{t+\Delta t}(\mathbf{x}) \}$$
(1)

where, [A] is a matrix and does not vary with time t.

An *m* set of time series data, $[X]_0^{(m-1)\Delta t}$, can be converted to $[X]_{\Delta t}^{m\Delta t}$ using [A],

$$[A] [X]_{0}^{(m-1)\Delta t}$$

= [[A] { $v_{0}(\mathbf{x})$ }, [A] { $v_{\Delta t}(\mathbf{x})$ },, [A] { $v_{(m-1)\Delta t}(\mathbf{x})$ }]
= [X] ${}_{\Delta t}^{m\Delta t}$ (2)

The DMD modes are eigenvectors of the matrix [*A*]. Instead of solving the eigenvalue problem of [*A*] directly, we used singular value decomposition (SVD), which saves time in solving the large-scale eigenvalue analysis and yields more robust results [10]. Using SVD, the matrix $[X]_0^{(m-1)\Delta t}$ can be decomposed into three matrices: [*U*], [Σ] and [*W*]:

$$[X]_0^{(m-1)\Delta t} = [U][\Sigma][W]^*$$
(3)

where, [U] and [W] are unitary matrices, and $[]^*$ is the transposed conjugate of each element.

By substituting equation (3) into equation (2), and after rearranging, the matrix [S] can be obtained:

$$[S] = [U]^*[A][U] = [U]^* [X]_{\Delta t}^{m\Delta t} [W][\Sigma]^{-1}$$
(4)

All matrices on the right-hand side of equation (4) can be calculated; hence, the matrix [*S*] can be determined. Since the matrix [*A*] contains the POD modes of the data sequence $[X]_0^{(m-1)\Delta t}$, the above operation amounts to a projection of the linear operator [*A*] onto a POD base. The eigenvectors of [*A*], the DMD modes, are calculated from the eigenvectors of the matrix [*S*] as:

$$\{\xi_r\} = [U]\{y_r\} \tag{5}$$

where, $\{\xi_r\}$ is the *r*-th DMD mode, and $\{y_r\}$ is the *r*-th eigenvector of [S], i.e., $[S]\{y_r\} = \kappa_r\{y_r\}$.

The data $\{v_{m\Delta t}(\mathbf{x})\}$ can be expressed as:

$$\{v_{m\Delta t}(\mathbf{x})\} = [A]^{m-1}\{v_0(\mathbf{x})\} = \sum_{r=1}^n c_r \, \kappa_r^{m-1}\{\xi_r\}$$
(6)

where, c_r is the coefficient determined for each mode with the dimensions of the specific physical quantity (velocity [m/s] in this study).

The coefficient c_r does not vary with time, but denotes the contribution of the *r*-th mode to the total flow pattern; only κ_r^{m-1} in equation (6) provides information related to time. The norm of κ_r determines the amplification of the flow structures in the DMD mode, while the characteristic frequency of the DMD mode can be obtained from the argument of κ_r as:

$$f_r = \arg[\kappa_r] / 2\pi / \Delta t \tag{7}$$

where, f_r is the frequency of the *r*-th DMD mode [Hz], and arg[] is the argument of complex number ($arg[] = tan^{-1}(Im()/Re())$).

Schmid determined the contribution rate c_r from the first time step Δt . However, some modes were quickly damped out and others were extremely divergent due to numerical errors. We proposed a new index c_{mid} [13][14] to clearly indicate the dominant mode:

$$c_{mid} = c_r \left| \kappa_r \right|^{T/2} \tag{8}$$

where, T is the data length.

 c_{mid_r} excludes the above-mentioned abnormal modes and it was confirmed that its square $(c_{mid_r})^2$ agreed well with the power spectral density distribution of velocity fluctuation in the wake of the square prism in smooth flow. In this study, some of the obtained DMD modes were excluded based on the c_{mid_r} , and the remaining modes were reordered by their characteristic frequencies.

Characteristic DMD Mode in the Separation Bubble

xz Plane

Because each element of a DMD mode was a complex number, the evolutional behavior of the mode with time was indicated by calculating the magnitude of each element and the phase difference from the element at the reference point (i.e., the element at the origin).



Figure 3. Vortex formation and convection in a characteristic period of dynamic mode decomposition (DMD) mode (u-component in xz plane).

Figure 3 shows the behavior of one of the characteristic DMD modes during a specific period for the u- component in the xz plane in smooth flow.

This mode was selected to accurately represent the timedependent image of the flow visualization. A clear discontinuity in magnitude and sign was recognized near the leading edge. This corresponded to the existence of a thin shear layer separated from the leading edge, which rolled up to a small vortex. The discontinuity possibly corresponded to the streamline binding the domain of the separation bubble, particularly near the leading edge. The small vortex increased in size during its movement downstream. The DMD mode represented the growth and convection of the vortex produced by the roll-up of the separating shear layer near the leading edge.

These flow patterns were similar to those reported by Bruno et al. [1], who observed significant differences between time-averaged and instantaneous flow patterns, and also that instantaneous reattachment occurred further upstream than in time-averaged flow. Although it was not easy to determine the reattachment point, the flow behind the downstream vortex in Figure 4 appears to be moving downward (negative *z* direction) toward the side surface, which implies that instantaneous reattachment occurred at roughly x/D = 1.6-2.4.



Figure 4. Instantaneous flow image near the leading edge in the xz plane, showing the formation of two clockwise-rotating vortices. Downward flow can be seen behind the downstream vortex (xz plane, in smooth flow).



Figure 5. Dominant DMD mode with a lower characteristic frequency than that shown in Figure 3, in which pulsating behavior in the magnitude of the separating shear layer near the leading edge and the following dissipated domain can be seen (xz plane, in smooth flow).

Figure 5 shows another typical mode for the *u*-component. In contrast to Figure 3, circulatory flow is not visible, but throbbing behavior can be seen in the separating shear layer near the leading edge and in the dissipated domain that follows. This corresponds to elongation and shrinkage of the separation bubble with a characteristic frequency of 2.04Hz.

When the turbulence was contained within the oncoming flow, the overall flow structure did not differ significantly from the structures in the smooth flow, except that the instantaneous reattachment was shifted upstream.

xy Plane

Figure 6 displays the DMD mode in smooth flow with a characteristic frequency similar to that of the mode shown in Figure 3. Areas with the same magnitude were located in parallel along the span-wise direction. A pair of areas with opposing signs shows the formation of a vortex roll, which displayed rotation along the *y* axis as it moved downstream with gradual dissipation. The maximum magnitude of the mode was located at approximately x/D = 1.0, between the first and second vortexes in Figure 4. This represents an identical flow structure to that in Figure 3.



Figure 6. DMD mode in the xy plane with a similar characteristic frequency to that of the mode shown in Figure 3 (*u*-component in smooth flow, distance from the side surface, h = 6.0 mm).

Figure 7 shows the same *u*-component DMD mode in smooth flow as displayed in Figure 6, except that the distance from the side surface is reduced to 4.0 mm. The area of high magnitude appears to have shifted downstream. The entire flow, in particular that near the leading edge, became more stagnant, when the distance from the visualized plane to the side surface *h* was < 4.0 mm.

However, if there was turbulence in the approaching flow, the flow in th xy plane became more irregular, as shown in Figure 8. The characteristic frequency of the displayed mode is the same as that in Figure 6. This suggests that there was also enhanced mixing in the separation bubble, and increased momentum transport in the span-wise direction.



Figure 7. DMD mode in the xy plane (u-component in smooth flow, distance from the side surface, h = 4.0 mm).



Figure 8. DMD mode in the *xy* plane (*u*-component in turbulent flow, distance from the side surface, h = 6.0 mm).

If the distance from the side surface h was reduced to 3 mm in turbulent flow, the entire flow was not stagnant but was still moving downstream, and an undulation along the span-wise direction was visible, as shown in Figure 9. The flow inside the separation bubble increased in turbulent flow. There was circulation with rotation around the *z*- axis of rotation, which increased the span-wise correlation.



Figure 9. DMD mode in the *xy* plane (*u*-component in turbulent flow, distance from the side surface, h = 3.0 mm).

Conclusion

The 2-D flow structures both inside and outside of a separation bubble near the leading edge of a 2-D flat rectangular section were visualized. The images were converted to time-dependent velocity vector fields using the PIV technique, and the timeseries data were analyzed using dynamic mode decomposition. The main findings from this study can be summarized as follows:

- 1) Vortex dominant-modes with convection and a pulsating mode in the entire separation bubble were captured in the *xz* plane in both uniform and turbulent flow.
- 2) An undulation mode was observed in the span-wise direction in the *xy* plane in turbulent flow.
- 3) These results imply a significant contribution to the increase in the span-wise correction of the flow structure.

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