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Evaluation of Mass and Damping Effects for Structures with Flexible Envelope and Dominant Opening

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Abstract

It has been well known that added mass effect should be taken into account in analysis of flexible structures. Moreover, damping properties are of great importance to understanding their structural performance in dynamics.

In this study, the mass and damping effects for the structures consisting of flexible envelope as well as a dominant opening is investigated theoretically as well as experimentally. These effects come mainly from air movement through the single dominant opening.

The structure is supposed to be a long-span structure that covers an enclosed space. The single dominant opening can be a door or a window that is open in service or broken due to high winds; it can also be considered as the lumped effect of the background leakage due to small openings or porosity on the building envelope. The experimental results are derived from free vibration tests.

Introduction

Mass and damping effects due to a dominant opening are of great importance to dynamic performance of the structures consisting of flexible envelope. In this study, we study these effects theoretically as well as experimentally.

For the purpose of indoor sports as well as events, more and more long-span structures, e.g., sports arenas, have been constructed all over the world. Many of these structures are covered facilities, and they require sufficient openings to provide ventilation and to allow for the movement of incoming and departing crowds.

The openings influence the dynamic properties of the roofs due to the leakage of the enclosed air, and also due to the associated changes in internal pressure [1]. When air moves inside and out of the covered structures through the openings, energy is consumed accordingly, resulting in damping effects. On the other hand, the roof of a long-span structure is usually flexible, and its deformation would also change the internal volume, and therefore, lead to fluctuations of internal pressure.

Recently, attempts have been devoted to constructing databases for multi-story buildings, in order to make rough estimates of damping ratio for these structures with respect to size, materials, and etc. For long-span structures, experimental measurements and analyses of several shell and spatial structures have been carried out by some researchers [2], although there is still very little real data available.

Furthermore, there is little existing studies on the mass and damping effects due to movement of air through the openings. This motivates our current study on the structures with flexible envelope and a single (lumped) dominant opening. In the paper by Novak and Kassem [1], the damping effect due to opening is assumed to be mainly caused by accoustical damping. They conducted experimental tests to verify their assumptions with good agreement, by using lound speaker for excitations. In the current study, we will demonstrate that linear damping is a reasonable assumption, however, acoustical damping is in fact insignificant compared to the energy loss due to movement of air through the opening.

The investigation is conducted by using experimental tests associated with theoretical studies for a simplified analysis model. Moreover, we limit us to a (scaled) small test model, rather than considering any real structures.

Governing Equations

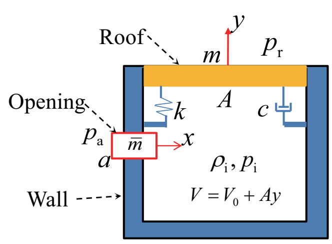


Figure 1: A 2DOF analysis model with a dominant opening and an elastically-mounted roof.

2DOF system

Consider a nominally-sealed structure with a dominant opening, that might be due to damage of a door or a window in high winds. A schematic of the analysis model is shown in Fig. 1. The analysis model consists of two degrees of freedom:

- The air in the opening is assumed to act like an air slug, moving (horizontally) in and out of the building model; (horizontal) displacement of the air slug is denoted by x.
- The flexible envelope is represented by an elastically-mounted roof that can move vertically, where the (vertical) displacement of the roof is denoted by y.

Denote the density of air in the air slug as ρ_a, its effetive length as l_e, the area of opening as a. Mass of the air slug is then given as $\bar{m} = a\rho_a l_e$.

Due to the fluctuation of external pressure p_w at the opening and internal pressure p_i, the motion equation for the air slug can be written as [3]

$$\bar{m}\ddot{x} + \bar{c}\dot{x} = (p_w - p_i)a, \tag{1}$$

and $\bar{c}$ is the (linearized) damping coefficient.

For the 2:1 rectangular opening as for the case in this study, the effetive length l_e of the air slug varies with shape and length of the opening [4]

$$l_e = 1.0 l_0 + 0.65\sqrt{a}, \tag{2}$$

where l_0 is the orifice length or thickness of the opening.

Let m denote the mass of the roof, A denote its area, and p_r denote the area-averaged external pressure acting on the roof. The governing equation for the roof due to difference of the internal pressure p_i and the external pressure p_r is then given as

$$m\ddot{y} + c\dot{y} + ky = (p_i - p_r)A, \tag{3}$$

where c is the damping coefficient, including the effects of structrual damping as well as acoustical damping, and k is the (spring) stiffness supporting the roof.

Let ρ_i denote density of the air inside the structure. From conservation of mass, we have

$$(\rho_a a)\dot{x} = \frac{d(\rho_i V)}{dt} = \rho_i \frac{dV}{dt} + V\frac{d\rho_i}{dt}, \tag{4}$$

where V is the internal volume of the structure given as

$$V = V_0 + Ay. \tag{5}$$

In equation (5), V_0 is the nominal internal volume (at $y = 0$).

From the isentropic law for air, we have

$$\frac{p_i}{\rho_i^\gamma} = \frac{p_0}{\rho_0^\gamma}, \tag{6}$$

where $\gamma(= 1.4)$ is the ratio of specific heats of air (or heat capacity ratio), $p_0(= 1.0\times 10^5 \mathrm{N/m^2})$ and $\rho_0(= 1.25\mathrm{kg/m^3})$ are the static pressure and the corresponding density (of air), respectively.

From equations (4)∼(6), and by using the (approximately) incompressible flow assumption, we have

$$p_i = \frac{\gamma p_0}{V_0}(ax - Ay) + p_0. \tag{7}$$

Using the internal pressure p_i derived in equation (7), the governing equation for the air slug in equation (1) becomes

$$\bar{m}\ddot{x} + \bar{c}\dot{x} + \frac{\gamma a^2 p_0}{V_0}x - \frac{\gamma A a p_0}{V_0}y = (p_w - p_0)a, \tag{8}$$

and the governing equation for the roof in equation (3) becomes

$$m\ddot{y} + c\dot{y} + (k + \frac{\gamma A^2 p_0}{V_0}y) - \frac{\gamma A p_0}{V_0}ax = (p_0 - p_r)A. \tag{9}$$

We denote

$$\omega_H = \sqrt{\frac{\gamma a p_0}{V_0 \rho l_e}}, \quad \omega_R = \sqrt{\frac{k}{m}}, \quad \omega_P = \sqrt{\frac{\gamma A^2 p_0}{m V_0}}, \tag{10}$$

where ω_H is the undamped Helmholtz resonance circular frequency for a corresponding rigid building with a dominant opening, ω_R is the undamped structural circular frequency of the elastically-mounted roof, and ω_P is the circular frequency associated with the pneumatic stiffness of the roof with respect to the contained air. The circular frequencies $\omega_{1,2}$ for the undamped coupled system are given as

$$\omega_{1,2} = \sqrt{\frac{\omega_H^2 + \omega_R^2 + \omega_P^2}{2} \mp \sqrt{\frac{(\omega_H^2 + \omega_R^2 + \omega_P^2)^2}{4} - \omega_H^2\omega_R^2}}. \tag{11}$$

Equivalent 1DOF System

From equation (7), we have

$$\dot{x} = \frac{V_0}{a\gamma p_0}\dot{p}_i + \frac{A}{a}\dot{y}, \quad \ddot{x} = \frac{V_0}{a\gamma p_0}\ddot{p}_i + \frac{A}{a}\ddot{y}. \tag{12}$$

By using equation (12), equations (1) and (3) can be combined into one equation as

$$M\ddot{y} + C\dot{y} + ky + \frac{V_0 A}{a^2\gamma p_0}(\bar{m}\ddot{p}_i + \bar{c}\dot{p}_i) = (p_w - p_r)A, \tag{13}$$

where

$$M = m + \frac{A^2}{a^2}\bar{m}, \quad C = c + \frac{A^2}{a^2}\bar{c}. \tag{14}$$

The pressure coefficient is defined as follows by using the reference wind speed U

$$C_{pi} = \frac{p_i - p_0}{\rho U^2/2}, \tag{15}$$

such that

$$\dot{C}_{pi} = \frac{\dot{p}_i}{\rho U^2/2}, \quad \ddot{C}_{pi} = \frac{\ddot{p}_i}{\rho U^2/2}. \tag{16}$$

Therefore, equation (13) can be rewritten as

$$M\ddot{y} + C\dot{y} + ky + \frac{V_0 A \rho U^2}{2a^2\gamma p_0}(\bar{m}\ddot{C}_{pi} + \bar{c}\dot{C}_{pi}) = (p_w - p_r)A \tag{17}$$

In free vibration tests, there is no wind; i.e., $U \approx 0$ and $p_w = p_r = p_0$, equation (17) can be simplified as follows with only one degree-of-freedom:

$$M\ddot{y} + C\dot{y} + ky = 0. \tag{18}$$

It is notable in equation (18) that the equivalent mass as well as damping effect of the whole system increase due to the coupling effect of the structure and the flow through opening.

The (approximated) natural circular frequency of the undamped system is given as

$$\bar{\omega} = \sqrt{\frac{K}{M}}. \tag{19}$$

Experiment Setup

In this section, we present the details on evaluation of the simplified 1DOF system, by using free vibration tests of a (scaled) test model presented in the previous section.

Test model

The test model is a 400×250×125mm^3 box as shown in figure 2. The roof and the walls are all rigid; i.e., stiff enough. The roof is supported by springs attached on the top of the walls. In this way, the roof can move vertically subjected to external loads. Note here that only the vertical translation motion of the roof changes the internal volume, and therefore, induces fluctuation of internal pressure.

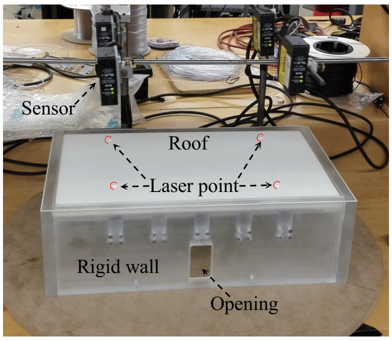


Figure 2: Test model.

Thickness of the wall is 17mm. The internal volume V_0 of the test model is $8.93\times10^{-3}\mathrm{m}^3$. The model is sealed except for the rectangular opening in the wall; moreover, the gaps between the roof and the walls are sealed by using latex. The size of the (dominant) opening is $30\times60\mathrm{mm}^2$; i.e., a = $1.8\times10^{-3}\mathrm{m}^2$, such that the open-area-to-wall ratio is 3.6%. This opening area is less than 5% of the windward surface, and it guarantees that equation (1) is applicable [3].

The effective mass m of the roof is 260g. Stiffness of the springs k is identified as 1.0657×10^3N/m in the static loading tests in our previous study [6].

Four laser sensors are used to measure displacements of the roof at its four corners. The sampling rate for the measurements is set as 200Hz. Average of the displacements measured by the four laser sensors are taken as the (mean vertical) displacement responses of the roof, also for the purpose of ruling out the rotation motions of the rigid roof.

Free vibration tests

In the free vibration tests, short shocks are applied to the roof. Moreover, two sets, five mass cases in each set, of free vibration tests have been conducted:

1. Free vibration tests in still air; i.e., the model is unattached from the bottom plate, and therefore, it has an infinite internal volume. Additional mass is placed on the top of the roof to change its mass m. There are in total five cases for the additional mass: +0g, +50g, +100g, +150g, and +200g.
2. Free vibration tests with the test model sealed except for the opening. There are also five cases for the additional mass placed on the top of the roof: +0g, +50g, +100g, +150g, and +200g.

For the system with one single generalized degree of freedom in free vibration tests, the ratio of two peaks y_i and y_{i+j} of the displacement responses can be expressed as [7]

$$\frac{y_i}{y_{i+j}} = \exp\left(\frac{2j\pi\xi}{\sqrt{1-\xi^2}}\right) \tag{20}$$

where ξ is the damping ratio of the generalized system. From equation (20), we have

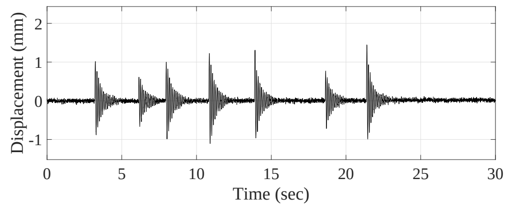
$$\xi = \sqrt{\frac{\delta^2}{4j^2\pi^2+\delta^2}}, \quad \text{with } \delta = \ln\left(\frac{y_i}{y_{i+j}}\right). \tag{21}$$

The damping ratio ξ and damping coefficient C has the following relationship

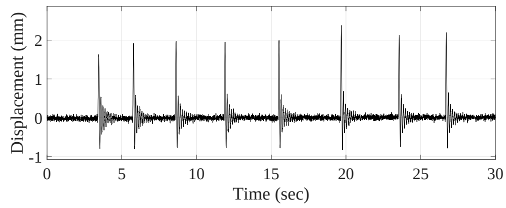
$$C = 2M\bar{\omega}\xi. \tag{22}$$

Results and Evaluation

Frequencies



(a) in still air



(b) with opening

Figure 3: Displacements of the roof in free vibration tests.

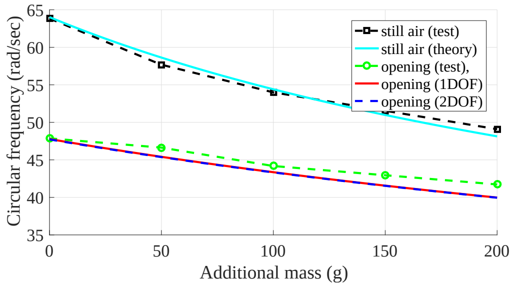


Figure 4: Circular frequencies.

Figure 3 shows the examples of displacement responses in time domain of the roof in free vibration tests: the example in (a) is for the case the the roof is placed in still air, and the other example in (b) is for the nominally sealed case with an opening.

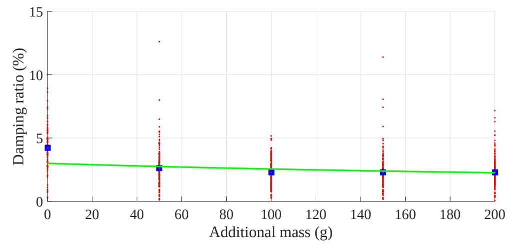
Figure 4 shows the natural circular frequencies identified from free vibration tests as well as theory. The natural circular frequencies in the tests, in still air as well as with opening, are identified by picking peak of the displacement responses in frequency domain. The frequencies from theory are calculated by using $\sqrt{m/k}$ for the elastically-mounted roof in still air, the formulations for the case associated with opening in equation (10) for the 2DOF system, and equation (19) for the equivalent 1DOF system.

It can be observed from figure 4 that the frequencies estimated by the theory agree well with those obtained from the free vibration tests. This can partially verify that the proposed formulations are correct.

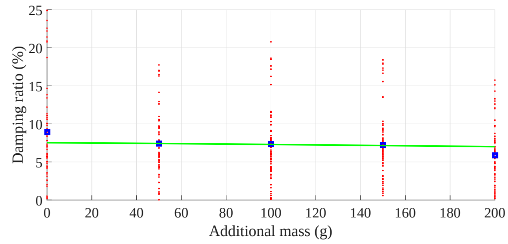
Moreover, the frequencies ω_1 estimated by equation (10) for the 2DOF system are almost identical to those $\bar{\omega}$ by equation (19) for the equivalent 1DOF system. This demonstrates that it is reasonable to ignore the inertial influence $\ddot{C}_{pi}$ coming from the fluctuation of internal pressure. However, this assumption in high winds should be verified in the future study.

Furthermore, the natural periods in all cases are elongated; i.e., the circular frequencies become smaller, while the opening is taken into consideration. This comes from the fact that mass of the system increases by $\bar{m}A^2/a^2 (= 206.9\text{g})$, which is comparative to mass of the roof.

Damping



(a) in still air



(b) with opening

Figure 5: Damping ratios identified in free vibration tests.

The band-pass filter is applied to exclude possible noises in the measurements. The signal below 0.5Hz as well as above 40.0Hz is cut off. The damping ratios calculated by application of equation (21) with $j = 1$ for successive peaks are plotted as red dots in figure 5 for each case. Their mean values are plotted as blue squares in the figure.

The green lines in figure 5 are the damping ratios estimated by using equation (22), with $c = 1.0$ for the cases in still air and $C = 4.5$ ($\bar{c} = 1.7 \times 10^{-3}$) for the case with opening. It is obvious from the figure that more energy losses, while the air moves inside and out of the test model through the opening, result in higher damping ratios (effects).

For the cases in still air as shown in figure 5(a), it can be observed that the estimated damping ratios agree well with their identified values in the tests, except for the case with no additional mass; i.e., the case with +0g. This shows that assumption of viscous damping for the elastically-mounted roof is reasonable.

Moreover, the higher damping for the case withough additional mass could be caused by the sealing problem: the gaps between the roof and the surrounding walls are sealed by latex, which is thin and have small bending stiffness. The top of the roof is designed to be on the same level as the top of the walls when it is not loaded. When the roof is loaded by the additional mass on the top, the bending in the latex is reduced, and therefore, its damping effect becomes smaller.

For the cases with opening as shown in figure 5(b), it is also demonstrated that viscous damping in the equivalent 1DOF system is also a reasonable assumption. For the acoustical damping as adopted in [1], the damping coefficient $\bar{c}$ is evaluated in the order of 10^{-6}, which is significantly smaller than that in the current study ($\bar{c} = 1.7 \times 10^{-3}$). This might comes from the fact that free vibration tests have been conducted in [1] by using loudspeaker, for which case the acoustical damping is dominant; however, this might not be applicable for usual cases.

Conclusions

Damping properties are of great importance to the structures in dynamics. This is especially the case for the long-span structures with flexible envolopes, which are more sensitive to vibration in comparison to other structural forms. This is because that they are usually flexible and their structural damping is small. Furthermore, the added mass effect is necessarily to be taken into account for this kind of structures.

In this study, we theoretically as well as experimentally investigated mass and damping effects, of a coupled system with a covered space as well as a (lumped) dominant opening.

It has been shown that the added mass effect can be accurately evaluated by a simplified 1DOF system. Moreover, for the test model, it has also been demonstrated that the damping effect can be evaluated as linear viscous damping. The acoustical damping was shown to be insignificant in our study.

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References

[1] Novak, M. & Kassem M., Effect of leakage and acoustical damping on free vibration of light roofs backed by cavities. *Journal of Wind Engineering and Industrial Aerodynamics*, **36**, 1990, 289-300.

[2] Shingu K., Hiratsuka K., Kondo N. & Irie T., Damping evaluation of spherical pre-stressed concrete shell by filtering and half-power bandwidth method, in *IASS Annual Symposium*, Tokyo, Japan, Sep., 2016.

[3] Holmes J.D., Mean and fluctuating internal pressures induced by wind, in *Proceedings of 5th Internal Conference on Wind Engineering*, Fort Collins, July, 1979, 435-450.

[4] Vickery B.J., Gust-factors for internal-pressures in low rise buildings. *Journal of Wind Engineering and Industrial Aerodynamics*, **23**, 1986, 259-271.

[5] Oh J.H., Kopp G.A. & Inculet D.R., The UWO contribution to the NIST aerodynamic database for wind loads on low buildings: Part 3. Internal pressures, *Journal of Wind Engineering and Industrial Aerodynamics*, **95(8)**, 2007, 755-779.

[6] Zhang J.Y. & Kopp G.A., A 2dof analysis model for vibration of flexible envelope with a dominant opening and its verification, in *8th International Colloquium on Bluff Body Aerodynamics and Applications*, Boston, USA, June, 2016.

[7] Chopra A.K., *Dynamics of Structures*. New Jersey: Prentice Hall, 1995.