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The effect of bridge vibration on the buffeting forces of flat closed-box bridges

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Abstract

Force-balance measurements of aerodynamic lifts and moments on flat closed-box bridge deck strips of a spring-suspended sectional model (SSSM) were carried out in a turbulent flow. Measurements of buffeting forces in the same sectional model without motion were also conducted for comparison. A new method of extracting buffeting lifts and moments of SSSM was proposed. The influence of bridge vibration on the spectra of buffeting lift and moment coefficients of bridge deck mainly occurs in the frequency region near the natural frequencies of SSSM.

1. Introduction

Aerodynamic forces on bridge deck in a turbulent flow are commonly separated into self-excited and buffeting force components. An experimental evaluation of all aerodynamic derivatives and aerodynamic admittances using wind tunnel models is still considered to be a most accurate means of estimating the unsteady aerodynamic forces and attendant response of long-span bridges. Aerodynamic admittances of bridge deck are usually identified in a turbulent flow through one algorithm based on tested fluctuating forces and wind velocities in a motionless sectional model. The study of influence of bridge vibration on the buffeting force of bridge deck is rare, and few researchers measure buffeting lifts and moments of SSSM using force balances.

In this study, non-wind-induced additional mass and damping coefficient due to model vibration in still air were extracted via free decay tests under zero wind speed, and they are supposed to be constant in all the testing cases with zero and non-zero wind speeds. Aerodynamic derivatives of bridge deck were then identified with the modified least square method under a range of wind speeds (Ding et al. [1]). The spectra of buffeting lift and moment coefficients of bridge deck under stochastic vibration were finally obtained through a new method using these parameters, and they were also compared with that directly measured from the motionless sectional model.

2. Experimental setup

A flat closed-box sectional model in the TJ-2 Wind Tunnel at Tongji University (China) was used to study the influence of stochastic vibration on the buffeting forces of bridge deck. The sectional model was constructed at a geometric scale of 1:45 and the width (*B*) and depth (*D*) of the model were 775.6 mm and 77.8 mm respectively with a width-over-depth ratio of B/D=9.71. Two inner walls were installed and fixed in the working section of the wind tunnel with a spacing of 1.8 m, and the length of model (*L*) was set to 1.74 m. The scheme of experimental setup in the turbulence flow is given in figure 1. Both sides of the sectional model were connected to supporting arms through rigid central shafts. Each supporting arm was hung by four pre-tensioned helical springs. The supporting system, including eight springs and two supporting arms, was placed inside the inner walls to ensure two-dimensionality of the flow and to avoid the end effect. The supporting system allowed the sectional model vibrating in vertical and torsional directions. The natural frequencies of SSSM in vertical and torsional directions were 1.594 Hz and 4.293 Hz, respectively. The designed mass and moment of inertia of the rigid sectional model were 11.139 kg/m and 0.426 kgm^2/m respectively.

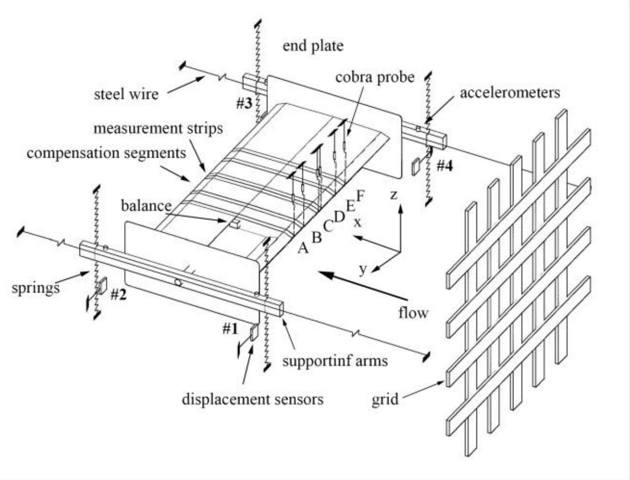

Figure 1. Scheme of experimental setup in the turbulence flow

To obtain the time-histories of oscillation curves of SSSM at a given wind speed in the turbulent flow, four accelerometers and four laser displacement sensors were mounted on each end of the two supporting arms. The distance of two sensors of the same kind on each side was 1.0 m. Total forces on six measurement strips of SSSM (consisting of aerodynamic force, inertial force and non-wind-induced force) were measured simultaneously with six independent small high frequency force balances. The 30-mm measurement strip was composed of three internal hollow layers (8 mm, 10 mm and 8 mm thick) and two 2-mm thick external layers, and the strip mass and moment of inertia were only about 0.061 kg and 0.00226 kgm^2 respectively. The strip mass and moment of inertia were very light compared with the designed ones of the whole model to reduce the ratio of inertial forces to total forces. Also, wind speeds above the measurement strips were also measured simultaneously with cobra probes. All signals were sampled at a frequency of 200 Hz. Further details concerning force balances, measurement strips, cobra probes, the turbulence wind field and the motionless sectional model are given by Yan et al. [3].

The responses of SSSM were captured by the sensors, and the vertical and torsional responses of displacement and acceleration can be respectively calculated by

$$h=\frac{x_1+x_3}{2},\alpha=\frac{x_1-x_2}{2l}, \tag{1}$$

$$\ddot{h}=\frac{y_1+y_3}{2},\ddot{\alpha}=\frac{y_1-y_2}{2l}, \tag{2}$$

where x_1, x_2 and x_3 are the measurements of laser displacement sensors 1, 2 and 3, respectively; y_1, y_2 and y_3 are the measurements of accelerometers 1, 2 and 3, respectively; $2l$ is the space between transducer 1 and transducer 2 and it was set to be 1.0 m in this study.

3. Wind tunnel test results

3.1 Non-wind-induced force due to model vibration in still air

The interaction between vibrating model and surrounding still air causes the non-wind-induced force, which can be taken into account introducing the additional mass and damping coefficient m_0 and c_{a1} in bending or the additional moment of inertia and damping coefficient I_0 and c_{a2} in torsion. These four parameters can be identified via the vertical or torsional free decay tests under zero wind speed as follows

$$\begin{aligned} L^0_{measured}(t) &= m_e\ddot{h}(t)+c_{e1}\dot{h}(t) \\ &= (m_s+m_0)\ddot{h}(t)+c_{a1}\dot{h}(t), \end{aligned} \tag{3a}$$

$$\begin{aligned} M^0_{measured}(t) &= I_e\ddot{\alpha}(t)+c_{e2}\dot{\alpha}(t) \\ &= (I_s+I_0)\ddot{\alpha}(t)+c_{a2}\dot{\alpha}(t), \end{aligned} \tag{3b}$$

where $L^0_{meausred}(t)$ and $M^0_{meausred}(t)$ are the measured total lift and moment on measurement strip under zero wind speed respectively; m_e and I_e are the equivalent mass and moment of inertia of measurement strip separately; c_{e1} and c_{e2} are the equivalent damping coefficients of measurement strip in bending and torsion respectively; m_s and I_s are the mass and moment of inertia of measurement strip separately; $\ddot{h}(t)$ and $\ddot{\alpha}(t)$ are the vertical and torsional responses of acceleration of SSSM measured by accelerometers respectively; $\dot{h}(t)$ and $\dot{\alpha}(t)$ are the vertical and torsional responses of velocity of SSSM coming from laser displacement sensors through a high-order finite-difference method separately.

With the measured total force histories on measurement strip and the response histories of acceleration and velocity of SSSM, the equivalent mass and damping coefficient can be identified through the time domain least square fitting method according to equation (3). Figure 2 and figure 3 show the measured total forces on Strip D in vertical and torsional free decay vibrations under zero wind speed respectively. To minimize the identification error, the identification of m_e, I_e, c_{e1} and c_{e2} should be conducted several times. The equivalent mass and moment of inertia of strip D are 0.1296 kg and 0.0223 kgm^2 respectively; the equivalent damping coefficients of Strip D in bending and torsion are 0.3611 $\mathrm{N\cdot s/m}$ and 0.0124 $\mathrm{Nm\cdot s/rad}$ respectively. The additional mass of Strip D is 0.0686 kg and occupies about 53% of the equivalent mass, and must be taken into consideration. Compared to the additional mass, the additional moment of inertia of Strip D is little in this study.

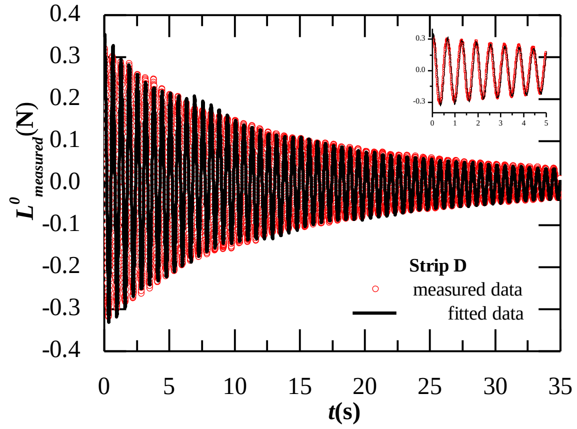

Figure 2. Measured total lift on Strip D in a vertical free decay vibration under zero wind speed

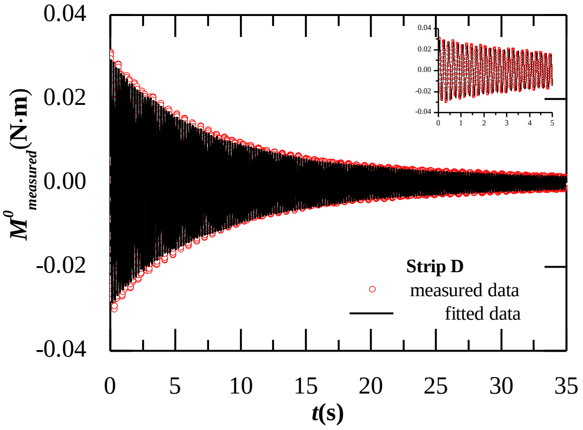

Figure 3. Measured total moment on Strip D in a torsional free decay vibration under zero wind speed

3.2 Aerodynamic force under non-zero wind speed

After identifying the equivalent mass and damping coefficient of measurement strip in bending and torsion, the aerodynamic lift and moment on measurement strip under non-zero wind speed can be given as follows

$$L_{wind}(t)=L_{measured}(t)-(m_e\ddot{h}(t)+c_{e1}\dot{h}(t)), \tag{4a}$$

$$M_{wind}(t)=M_{measured}(t)-(I_e\ddot{\alpha}(t)+c_{e2}\dot{\alpha}(t)), \tag{4b}$$

where $L_{wind}(t)$ and $M_{wind}(t)$ are the aerodynamic lift and moment on measurement strip at a given wind speed respectively; $L_{measured}(t)$ and $M_{measured}(t)$ are the measured total lift and moment on measurement strip at a given wind speed respectively.

3.3 Aerodynamic derivative

The self-excited lift and moment acting on unit span of SSSM can be expressed as a linear function of nodal displacement and nodal velocity (Scanlan and Tomko [2])

$$L_{se}(t)=\frac{1}{2}\rho U^2B\left[KH_1^*\frac{\dot{h}(t)}{U}+KH_2^*\frac{B\dot{\alpha}(t)}{U}+K^2H_3^*\alpha(t)+K^2H_4^*\frac{h(t)}{B}\right], \quad (5a)$$

$$M_{se}(t)=\frac{1}{2}\rho U^2B^2\left[KA_1^*\frac{\dot{h}(t)}{U}+KA_2^*\frac{B\dot{\alpha}(t)}{U}+K^2A_3^*\alpha(t)+K^2A_4^*\frac{h(t)}{B}\right], \quad 5b)$$

where ρ is air mass density; U is mean wind speed; B is width of the bridge deck; f is engineering frequency; $K = 2\pi fB/U$ is reduced circular frequency; H_i^* and A_i^* (i=1,2,3,4) are aerodynamic derivatives that are expressed in terms of reduced wind velocity *U/fB* and determined by the geometry configuration of the deck's cross section theoretically.

The aerodynamic derivatives of the flat closed-box deck in the turbulence flow were identified through the modified least square method (Ding et al. [1]). The test results of aerodynamic derivative of bridge deck are given in figure 4.

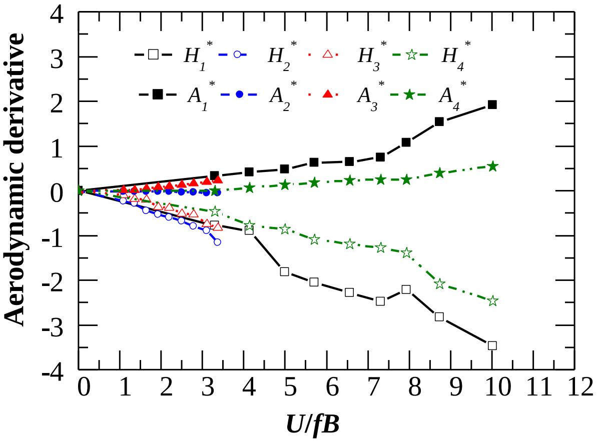

Figure 4. Aerodynamic derivative of bridge deck in the turbulence flow

3.4 *Spectra of buffeting force coefficient*

The self-excited forces on unit span in the lift and moment directions L_{se} (t) and M_{se} (t) respectively, can be rendered dimensionless and expressed in terms of self-excited lift and moment coefficients $C_{L_{se}}(t)$ and $C_{M_{se}}(t)$ as

$$C_{L_{se}}(t)=L_{se}(t)/0.5\rho U^2B, \quad (6a)$$

$$C_{M_{se}}(t)=M_{se}(t)/0.5\rho U^2B^2. \quad (6b)$$

Denoting

$$H_{\dot{h}}=\frac{KH_1^*}{U}, H_{\dot{\alpha}}=\frac{KH_2^*B}{U}, H_\alpha=K^2H_3^*, H_h=\frac{K^2H_4^*}{B}, \quad (7a)$$

$$A_{\dot{h}}=\frac{KA_1^*}{U}, A_{\dot{\alpha}}=\frac{KA_2^*B}{U}, A_\alpha=K^2A_3^*, A_h=\frac{K^2A_4^*}{B}, \quad (7b)$$

then equation (6) is rewritten as

$$C_{L_{se}}(t)=H_{\dot{h}}\dot{h}(t)+H_{\dot{\alpha}}\dot{\alpha}(t)+H_\alpha\alpha(t)+H_hh(t), \quad (8a)$$

$$C_{M_{se}}(t)=A_{\dot{h}}\dot{h}(t)+A_{\dot{\alpha}}\dot{\alpha}(t)+A_\alpha\alpha(t)+A_hh(t). \quad (8b)$$

Since the self-excited lift and moment coefficients of SSSM $C_{L_{se}}(t)$ and $C_{M_{se}}(t)$ are actually the functions of time t and frequency f, they can be converted into the forms of Fourier Transform as

$$C_{L_{se}}(f)=H_{\dot{h}}\dot{h}(f)+H_{\dot{\alpha}}\dot{\alpha}(f)+H_\alpha\alpha(f)+H_hh(f), \quad (9a)$$

$$C_{M_{se}}(f)=A_{\dot{h}}\dot{h}(f)+A_{\dot{\alpha}}\dot{\alpha}(f)+A_\alpha\alpha(f)+A_hh(f), \quad (9b)$$

where $\dot{h}(f)$ and $\dot{\alpha}(f)$ are the forms of Fourier Transform of vertical and torsional velocities of SSSM respectively; $h(f)$ and $\alpha(f)$ are the forms of Fourier Transform of vertical and torsional displacements of SSSM respectively.

On the other hand, the buffeting lift and moment coefficients of SSSM $C_{L_b}(t)$ and $C_{M_b}(t)$ in a turbulence flow can be given as follows

$$C_{L_b}(t)=C_{L_{wind}}(t)-C_{L_{se}}(t), \quad (10a)$$

$$C_{M_b}(t)=C_{M_{wind}}(t)-C_{M_{se}}(t), \quad (10b)$$

where $C_{L_{wind}}(t)$ and $C_{M_{wind}}(t)$ are the aerodynamic lift and moment coefficients of SSSM separately.

Equation (10) can then be rewritten in the forms of the Fourier Transform of buffeting lift and moment coefficients of SSSM as

$$C_{L_b}(f)=C_{L_{wind}}(f)-C_{L_{se}}(f), \quad (11a)$$

$$C_{M_b}(f)=C_{M_{wind}}(f)-C_{M_{se}}(f). \quad (11b)$$

After obtaining the Fourier Transforms of buffeting lift and moment coefficients of SSSM $C_{L_b}(f)$ and $C_{M_b}(f)$, the spectra of buffeting lift and moment coefficients of SSSM $S_{C_{L_b}}(f)$ and $S_{C_{M_b}}(f)$ can be obtained from the following relationships

$$S_{C_{L_b}}(f)=\left|C_{L_b}(f)\right|^2/N, \quad (12a)$$

$$S_{C_{M_b}}(f)=\left|C_{M_b}(f)\right|^2/N, \quad (12b)$$

where N is the length of sampled random signal sequence.

3.5 *Comparison of spectra of buffeting force coefficient*

Two separated tests focused on measuring the buffeting forces on measurement strips of the motionless and moving sectional model in a grid-generated turbulence at a wind speed of 8.0 m/s were carried out. The balance, wind speed, displacement and acceleration were all sampled at 200 Hz and 131072 data were acquired, corresponding to a period of 655 s. The piecewise smooth method was applied in the spectral analysis of each segment, which was further divided into 16 sub-segments with an overlapped length of half a sub-segment between two neighbouring sub-segments. There were 31 segments and the data point number of each segment for the FFT was equal to 8192. The hamming window was used in the spectral analysis of each-segment in order to decrease the leakage of the signals in the frequency domain.

The spectra of buffeting force coefficients on measurement strips of the motionless sectional model in the low reduced frequency region (*fB/U*<1) can be directly measured and fitted by

$$\log_{10}(S_{C_{F_b}}(K_B))=a_0+\frac{a_1+a_2K_B}{1+a_3K_B+a_4K_B^2}(F=L,M), \quad (13)$$

where $K_B = fB/U$; a_0, a_1, a_2, a_3, a_4 are determined by the fitting the equation to measured data.

The spectral analysis for self-excited force coefficients of SSSM has been performed for the low frequencies f under 10.6 Hz, which is corresponding to *fB/U* of 1.0. Aerodynamic derivatives in terms of reduced wind speed *U/fB* were fitted by quadratic polynomial functions firstly. The fitted aerodynamic derivatives were then converted to the form of reduced frequency *fB/U*, and the lower limit of frequency was set to be 0.2 Hz.

The Fourier Transforms of buffeting lift and moment coefficients on Strip D of SSSM based on the first segment of sampled data (41 s) are given in figure 5. The real and imaginary parts of buffeting force coefficient Fourier Transforms fluctuate dramatically within the whole reduced frequency region of interest, and there was a relatively huge flux for buffeting moment coefficient at the torsional natural frequency.

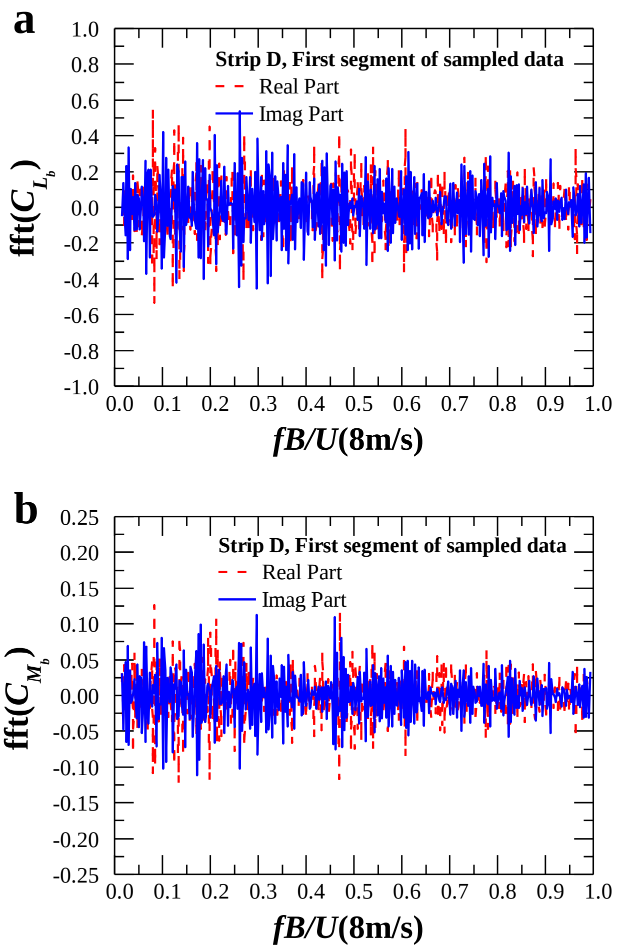

Figure 5. Fourier Transforms of buffeting force coefficients on Strip D of SSSM based on the first segment of sampled data

The spectra of buffeting lift and moment coefficients based on the 31 segments of sampled data were obtained using equation (12) respectively, and the spectra of buffeting force coefficients on Strip D of SSSM were the average results of these 31 spectra.

The spectra of buffeting force coefficient on Strip D of the motionless and moving sectional model are given in figure 6. The influence of bridge stochastic vibration on the spectra of buffeting force coefficients of flat closed-box bridge deck mainly occurs in the frequency region near the natural frequencies of the SSSM, especially for buffeting moment coefficient at the torsional natural frequency in this study.

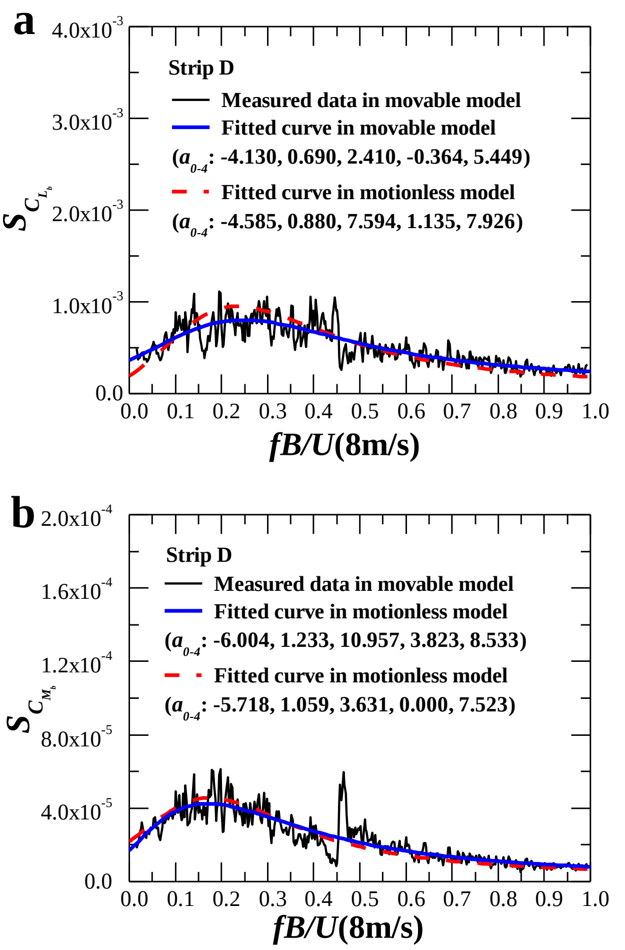

Figure 6. Spectra of buffeting force coefficients on Strip D of the motionless and moving sectional model

4. Conclusions

A new method of extracting buffeting lift and moment on measurement strip of SSSM using force balance was proposed. The effect of bridge vibration on the spectra of buffeting lift and moment coefficients of bridge deck mainly occurs in the frequency region near the natural frequencies of SSSM.

5. Acknowledgments

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