

## A Wind Load and Structural Parameters Estimation Approach for Building Structures

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### Abstract

This study presents a novel algorithm for simultaneous estimation of unknown wind load and structural parameters from wind-induced acceleration responses. It integrates an unbiased minimum variance input estimation and the recursive least squares estimation concept. The inverse identification approach includes four parts: time update, estimation of unknown wind load, measurement update and estimation of unknown parameters. A ten-story shear building is chosen as an example to validate the feasibility and accuracy of the proposed algorithm. Numerical simulation results indicate that the proposed method can be used to estimate wind load and structural parameters from wind induced structural acceleration responses accurately and effectively.

### Introduction

Many engineering applications require the knowledge of both wind load and structural parameters. However with the limitation of the measurement equipment, it is difficult or even impossible to directly measure the required excitations and parameters from prototype structures. Therefore it is of great interest to obtain an approach to estimate the unknown wind load and structural parameters. Structural parameters identification has been actively investigated in the past few decades, including frequency domain methods, time domain methods and time-frequency domain methods. A great deal of methods have been proposed for identifying civil structures such as least squares methods [16,20], Monte Carlo methods [4,9,12], wavelet transform based methods [2,6,17], Kalman filter methods and Hilbert transform based methods[5,19,15].

The wind load estimation is a crucial topic in structural wind engineering. Chen and Li [3] proposed a general statistical average algorithm to estimate unknown wind load. Kang and Lo [13] identified the wind load on an elevated tower based on the discretized governing equations. Law et al. [14] developed regularization method to obtain the unknown wind load based on structural displacement or strain responses. Hwang et al. [10,11] estimated the modal wind load based on Kalman filter method with limited measured responses. Zhi et al. [21,22] developed Kalman filter method to estimate the wind load on super-tall buildings with limited structural responses and the effects of crucial parameters such as the type of wind-induced response, the covariance matrix of noise, errors of structural modal parameters, and the number of vibration modes were studied.

It is obvious that the researches mentioned above only identify structural parameters or wind load. However, in practice, both structural parameters and wind load are difficult to be directly measured. Several studies have been performed to simultaneously estimating unknown excitations and structural parameters. Wang and Haldar [18] combined least squares method with the extended Kalman filter to identify structural parameters and unknown input. Yang et al. [20] combined an

analytical recursive solution based on the least squares estimation method with an adaptive tracking technique for identification of parameters and unknown input. Al-Hussein and Haldar [1] proposed an approach for parameters and unknown excitations estimation based on unscented Kalman filter. Although several studies identified the structural parameters without information of external excitations, most of the unknown input is earthquake excitation. The previous researches cannot be directly applied to wind load identification with unknown structural parameters.

In this study, an inverse approach for simultaneous identification of time-varying wind load and structural parameters is presented. The estimation of wind load is based on unbiased minimum variance input estimation [7,8] and structural parameters identification is on the basis of recursive least squares estimation concept. The feasibility of the proposed method is validated through numerical analysis of a ten-story shear building system subject to wind load. The aim of this study is to propose an effective approach for wind load identification with unknown structural parameters.

### Proposed algorithm

The second-order differential equation of motion of an n degrees of freedom building structure can be given by

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \quad (1)$$

in which  $M$ ,  $C$ ,  $K$  are mass, damping and stiffness matrices of the building structure, respectively.  $\ddot{x}(t)$ ,  $\dot{x}(t)$ ,  $x(t)$  are structural acceleration, velocity, and displacement response vectors, respectively.  $F(t)$  is the wind load vector.

The state vector consists of structural displacement, velocity can be written as

$$Z(t) = [\dot{x}(t) \ x(t)]^T \quad (2)$$

Then equation (1) can be expressed in state space at time  $t=(k+1)\Delta t$  as

$$Z_{k+1} = A_k Z_k + B_k F_k \quad (3)$$

where  $A_k = (I_{2n \times 2n} + \Delta t \begin{bmatrix} 0_{n \times n} & I_{n \times n} \\ -M^{-1}K & -M^{-1}C_k \end{bmatrix})$ ,  $B_k = \Delta t \begin{bmatrix} 0_{n \times n} \\ -M^{-1} \end{bmatrix}$ .

The damping matrix  $C_k$  means that it is the estimation of structural damping at time  $t=k\Delta t$ .

The measurement equation associated with equation (1) at time  $t=k\Delta t$  can be given as

$$y_k = D_k Z_k + G_k F_k + v_k \quad (4)$$

where  $y_k = \ddot{x}(k\Delta t)$  is the  $n \times 1$  measurement acceleration vector.  $D_k = [-M^{-1}K \ -M^{-1}C_k]$  and  $G_k = [-M^{-1}]$ .  $v_k$  is a  $n \times 1$  Gaussian

measurement noise vector with zero mean and covariance matrix, where  $\mathbf{R}_k \delta_k = E(\mathbf{v}_k \mathbf{v}_k^T)$  is the Kronecker delta.

Based on the unbiased minimum variance input estimation method [7,8], the state and input estimation can be calculated as follows

Time update:

$$\hat{\mathbf{Z}}_{k+1|k} = \mathbf{A}_k \hat{\mathbf{Z}}_{k|k} + \mathbf{B}_k \hat{\mathbf{F}}_k \quad (5)$$

$$\mathbf{P}_{Z,k+1|k} = \begin{bmatrix} \mathbf{A}_k & \mathbf{B}_k \end{bmatrix} \begin{bmatrix} \mathbf{P}_{Z,k|k} & \mathbf{P}_{ZF,k|k} \\ \mathbf{P}_{FZ,k|k} & \mathbf{P}_{F,k|k} \end{bmatrix} \begin{bmatrix} \mathbf{A}_k^T \\ \mathbf{B}_k^T \end{bmatrix} \quad (6)$$

Estimation of unknown wind load:

$$\tilde{\mathbf{R}}_{k+1} = \mathbf{D}_{k+1} \mathbf{P}_{Z,k+1|k} \mathbf{D}_{k+1}^T + \mathbf{R}_{k+1} \quad (7)$$

$$\mathbf{K}_{F,k+1} = (\mathbf{G}_{k+1}^T \tilde{\mathbf{R}}_{k+1}^{-1} \mathbf{G}_{k+1})^{-1} \mathbf{G}_{k+1}^T \tilde{\mathbf{R}}_{k+1}^{-1} \quad (8)$$

$$\hat{\mathbf{F}}_{k+1} = \mathbf{K}_{F,k+1} (\mathbf{y}_{k+1} - \mathbf{D}_{k+1} \hat{\mathbf{Z}}_{k+1|k}) \quad (9)$$

$$\mathbf{P}_{F,k+1} = (\mathbf{G}_{k+1}^T \tilde{\mathbf{R}}_{k+1}^{-1} \mathbf{G}_{k+1})^{-1} \quad (10)$$

Measurement update:

$$\mathbf{K}_{Z,k+1} = \mathbf{P}_{Z,k+1|k} \mathbf{D}_{k+1}^T \tilde{\mathbf{R}}_{k+1}^{-1} \quad (11)$$

$$\hat{\mathbf{Z}}_{k+1|k+1} = \hat{\mathbf{Z}}_{k+1|k} + \mathbf{K}_{Z,k+1} (\mathbf{y}_{k+1} - \mathbf{D}_{k+1} \hat{\mathbf{Z}}_{k+1|k} - \mathbf{G}_{k+1} \hat{\mathbf{F}}_{k+1}) \quad (12)$$

$$\mathbf{P}_{Z,k+1|k+1} = \mathbf{P}_{Z,k+1|k} - \mathbf{K}_{Z,k+1} (\tilde{\mathbf{R}}_{k+1}^{-1} - \mathbf{G}_{k+1} \mathbf{P}_{F,k+1} \mathbf{G}_{k+1}^T) \mathbf{K}_{Z,k+1}^T \quad (13)$$

$$\mathbf{P}_{ZF,k+1|k+1} = \mathbf{P}_{ZF,k+1|k+1}^T = -\mathbf{K}_{Z,k+1} \mathbf{G}_{k+1} \mathbf{P}_{F,k+1} \quad (14)$$

Then structural parameters can be estimated based on the recursive least squares estimation method with the updated state and unknown wind load at time  $t=(k+1)\Delta t$ .

Suppose  $\theta_{k+1}$  is the unknown structural damping at time  $t=(k+1)\Delta t$ . Then measurement equation for parameter estimation can be expressed as

$$\phi_{k+1} \theta_{k+1} = \hat{\mathbf{F}}_{k+1} - \mathbf{M} \mathbf{y}_{k+1} - \mathbf{K} \mathbf{x}_{k+1} \quad (15)$$

in which  $\phi_{k+1}$  is data matrix.

The recursive solution for  $\theta_{k+1}$  can be calculated as

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \mathbf{K}_{\theta,k+1} (\hat{\mathbf{F}}_{k+1} - \mathbf{M} \mathbf{y}_{k+1} - \phi_{k+1} \hat{\theta}_k - \mathbf{K} \mathbf{x}_{k+1}) \quad (16)$$

in which

$$\mathbf{K}_{\theta,k+1} = \mathbf{P}_{\theta,k} \phi_{k+1}^T (\mathbf{I}_{n \times n} + \phi_{k+1} \mathbf{P}_{\theta,k} \phi_{k+1}^T)^{-1} \quad (17)$$

$$\mathbf{P}_{\theta,k} = (\mathbf{I}_{2n \times 2n} - \mathbf{K}_{\theta,k} \phi_{k+1}) \mathbf{P}_{\theta,k-1} \quad (18)$$

## Numerical simulation

A ten-story shear building structure is chosen as an example to check the effectiveness of the algorithm. The height of each story is 3 m. The building has uniform mass  $m=125$  kg and the uniform horizontal story stiffness  $k=24.2$  kN/m for all stories. The structural damping is assumed to be viscous damping and set to be  $c=175$  N s/m of each story.

In this numerical simulation, the fluctuating wind speed is simulated according to auto-regressive model method and the spectral density used to simulate the fluctuating wind speed is Davenport spectrum. The vertical wind profile is taken as the power law and the exponent  $\alpha$  is 0.22. According to Chinese National Load Code the reference height is set to be 10 m and the mean wind speed at the reference height is 10 m/s. The simulated fluctuating wind speed on the 5<sup>th</sup> and 10<sup>th</sup> floor are shown in figure 1. The comparison of the power spectral density of the simulated fluctuating wind speed on the 5<sup>th</sup> and 10<sup>th</sup> floor are shown in figure 2. It can be seen that the simulated power spectrum density curve matches very well with Davenport spectrum. The density of air is assumed to be 1.23 kg/m<sup>3</sup>. The drag coefficient of the structure is set to be 1.3. The orthogonal exposed wind area of each floor is 50 m<sup>2</sup>.

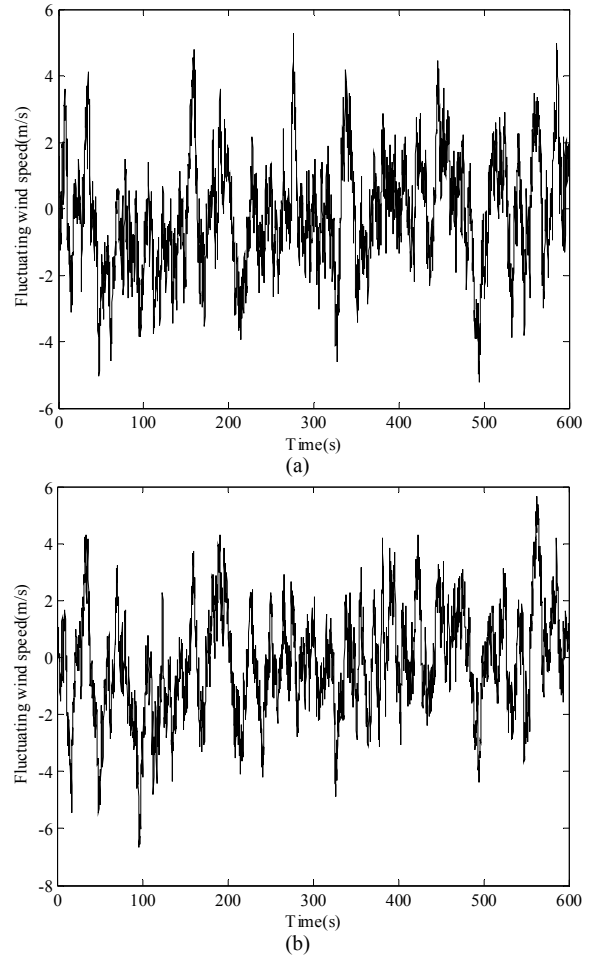


Figure 1. Simulated fluctuating wind speed. (a) On the 5<sup>th</sup> floor, (b) On the 10<sup>th</sup> floor.

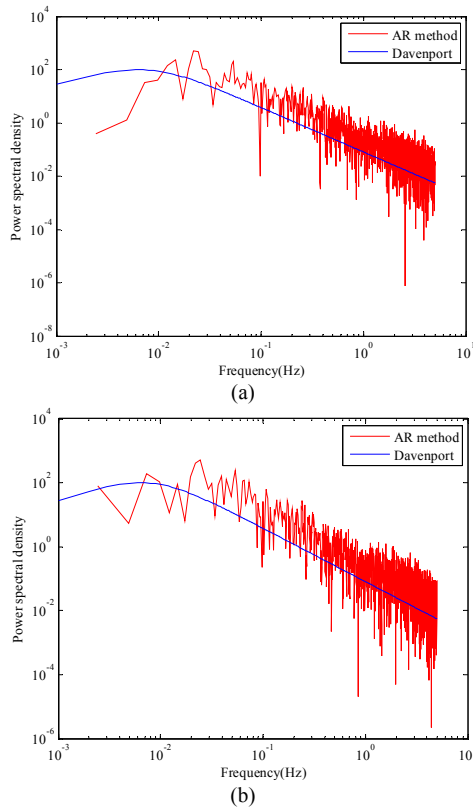


Figure 2. Comparison of power spectral density of fluctuating wind speed between simulated and Davenport spectra. (a) On the 5<sup>th</sup> floor, (b) On the 10<sup>th</sup> floor.

The measured structural acceleration responses are simulated from theoretically computed quantities based on Newmark- $\beta$  method and superimposed with 2% RMS white noise. The initial value and error covariance of state vector  $\mathbf{Z}_0$  are set to be  $[0,0,0,\dots,0,0,0]_{20 \times 1}$  and  $\mathbf{I}_{20 \times 20}$ , respectively. Unknown parameter to be identified is damping coefficient. The initial value and error covariance of unknown parameter are  $0.8c$  and  $5 \times 10^{-2}$ , respectively. The identified result of the damping coefficient is presented in figure 3 with dash-dot line and compared with the exact one with solid line. Clearly, the proposed method can identify constant structural damping coefficient very well. The comparison of the real and identified values of time-varying wind load on the 5<sup>th</sup> and 10<sup>th</sup> floor are shown in figure 4. In figure 4, the identified results are presented as dash-dot lines but the real values are presented as solid lines for comparison. It can be seen that the identified results are very close to the real values. The proposed method can identify the time-varying wind load accurately.

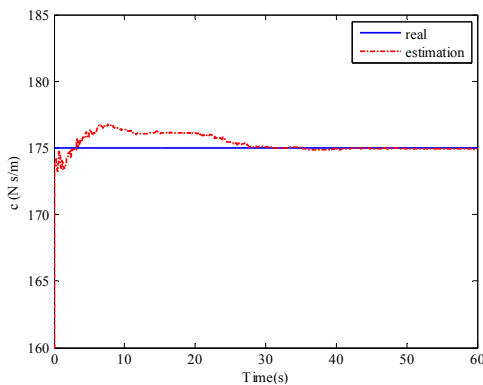


Figure 3. Comparison of structural damping coefficient between exact and estimated value.

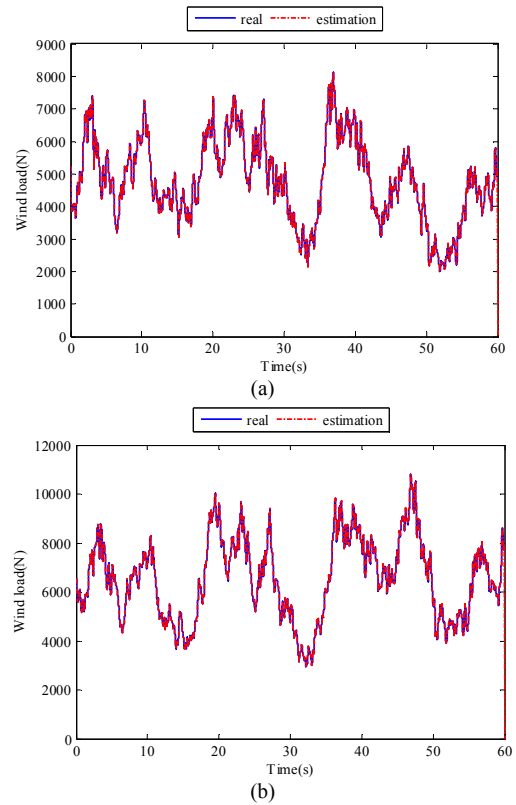


Figure 4. Comparison of time-varying wind load between exact and estimated values. (a) On the 5<sup>th</sup> floor, (b) On the 10<sup>th</sup> floor.

## Conclusions

An efficient method for simultaneous identification of time-varying wind load and structural parameters has been presented in this paper. The feasibility and accuracy of the proposed approach have been assessed through numerical simulation of a ten-story shear building structure. Comparative studies show that the identified results are very close to the real values. Results indicate that the proposed algorithm can be an effective approach for simultaneously identify unknown wind load and structural parameters.

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