9th Asia-Pacific Conference on Wind Engineering
Auckland, New Zealand
3-7 December 2017

Nonlinear buffeting response of inclined stay cables

L. Carassale[1], G. Piccardo[2]

[1]Department of Mechanical, Energy, Management and Transportation Engineering – DIME
University of Genoa, Genoa, Italy

[2]Department of Civil, Chemical and Environmental Engineering – DICCA
University of Genoa, Genoa, Italy

Abstract

Cable-supported structures have great diffusion in the modern engineering due to their ability to cover large spans. Among them, cable-stayed systems are increasingly used; typical examples are bridges and towers, with lengths exceeding 1 km. Cable response to wind actions seems very important for both the cables themselves and the vibrations that they can transmit to the supported structure (e.g., cable-deck interactions). There exist sophisticated mechanical models for the deterministic analysis of a single cable. The analysis of the wind-induced response of a cable to a turbulent wind is surely more uncertain. Furthermore, it is very challenging to study cable-stayed systems supported by many cables that are close to each other. In these cases, the fluid action on the cables can be greatly affected by their relative position due to the spatial structure of the atmospheric turbulence. The aim of this paper is contributing to this problem by proposing a simulation method of turbulent winds that can efficiently consider the possible correlations of the forces acting on different cables. The example proposed concerns a group of inclined cable stays, (relatively) close to each other, that simulate the support of a cable-stayed tower of great height.

Introduction

The formulation of sophisticated mechanical models (e.g., [1,2]) for the dynamic response of suspended cables in linear and non-linear regime has received a great impetus in the last decade. Meanwhile the ability to simulate wind velocity fields on large and complex systems has become possible as well (e.g., [3]). Since wind-tunnel experiments (e.g., [4]) point out the important role of the atmospheric turbulence (and hence of the randomness) in the cable response, the merging of the two research fields appears to be of interest. But, while their deterministic evaluation has reached an advanced stage of knowledge (e.g., [5]), the dynamics of randomly excited cables need further investigations (e.g., [6]). In particular, the probability distribution of the cable response can be significantly non-Gaussian due to nonlinearities of different nature (e.g., [7]). For this reason, the models employed in the analyses should contain all the relevant non-linear terms that may influence the probabilistic response of the cable and, consequently, they can be very burdensome. In this context, the formulation of advanced numerical techniques, such as improved nonlinear finite element method [8] and reduced-order model [9], seems necessary in order to reduce the computational effort.

The problem that presents many difficulties for a single cable becomes extremely costly (or almost impossible) to deal with a group of cable stays, (relatively) close to each other, that may induce correlation between actions on each cable. Simulations must be resolute and lengthy because frequencies and damping ratios in play are extremely small. The usual simulation schemes lead to enormous computational time. The actions thus obtained become practically unmanageable in commercial finite-element codes. The use of analytical cable models to apply the simulated actions becomes almost mandatory.

Starting from an analytical formulation of the cable dynamics, this paper proposes a simulation method of turbulent winds specifically aimed at group of cable stays, to take into account the possible correlation of the actions that involve them. The example proposed concerns a group of inclined cable stays simulating the support of a high-rise tower.

Pre-stressed model of an inclined cable

Considering the prestressed configuration as the reference (or initial) one, the use of relative positions allows some simplifications, such as series expansions around equilibrium. The response of a single stay cable to incremental loads is then performed in the current configuration using the hypothesis of shallow cables.

Initial configuration

The cable is modeled as a flexible axis line with planar, rigid cross sections. The axis line is a curve whose positions, at the initial time, lie on a vertical plane and are identified by $\bar{\mathbf{x}}(s)$, where s is the length parameter of the curve (i.e. its curvilinear abscissa), $s \in [0, L]$, and L is the cable length ($L \geq L_0$, L_0 being the chord length; Figure 1). The corresponding configuration is called initial or reference configuration, $\bar{C}$; the over-bar indicates quantities related to the initial configuration. A local reference Frenet triad is shown in Fig. 1, $\bar{\mathbf{a}}_1(s)$ being the tangent and $\bar{\mathbf{a}}_2(s)$ the normal versor.

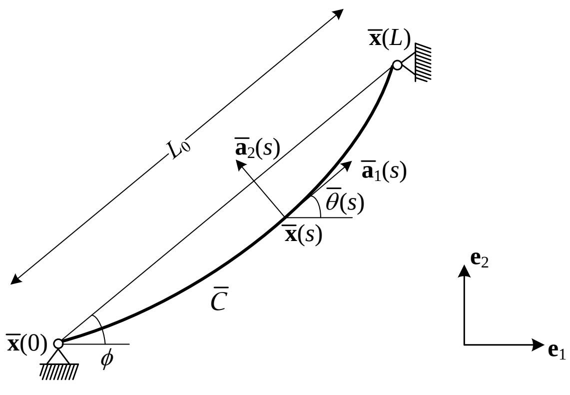

Figure 1. Inclined cable: chord and initial configuration.

The components of $\bar{\mathbf{x}}(s)$ on the canonical base for the inextensible catenary (i.e., neglecting the static stretching) are [10]:

$$
\begin{aligned}
\bar{x}(s) &= \frac{\bar{H}}{mg}\left[\operatorname{arcsinh}\left(\frac{mgs}{\bar{H}}+c\right)-\operatorname{arcsinh}c\right] \\
\bar{y}(s) &= \frac{\bar{H}}{mg}\left[\sqrt{1+\left(\frac{mgs}{\bar{H}}+c\right)^2}-\sqrt{1+c^2}\right]
\end{aligned} \tag{1}
$$

where m is the mass per-unit-length of the cable, g is the gravity acceleration, $\bar{H}$ is the constant horizontal stress (acting on the top of the cable). The axial stress of the cable is:

$$
\bar{T}(s) = \frac{\bar{H}}{\cos\bar{\theta}(s)}, \quad \bar{\theta}(s) = \arctan\left(\frac{mgs}{\bar{H}}+c\right) \tag{2}
$$

where $\bar{\theta}$ is the slope angle (Figure 1). Therefore, the initial curvature of the cable, defined as $\bar{\kappa}(s) = \theta'(s)$, assumes the following expression:

$$
\bar{\kappa}(s) = \frac{mg}{\bar{H}}\frac{1}{1+\left(\frac{mgs}{\bar{H}}+c\right)^2} \tag{3}
$$

Since the value of the stress $\bar{T}$ is usually prescribed for each cable at $\bar{\mathbf{x}}(L)$, i.e. $\bar{T}(L) = T_{design}$, the values of constant c and of the cable length L are numerically obtained by Eq. (1) imposing the boundary conditions, that is $\bar{x}(L) = L_0 \cos\phi$, $\bar{y}(L) = L_0 \sin\phi$, ϕ being the inclination of the cable (Figure 1). Therefore the stress $\bar{T}$, the slope $\bar{\theta}$ and the curvature $\bar{\kappa}$ are known for each value of the abscissa s.

Current configuration

The current configuration assumed by the inclined cable at the generic time $t > 0$ is identified as C (Figure 2). In this configuration, which is generally non planar, the cable is considered as a polar continuum excited by dynamic wind forces.

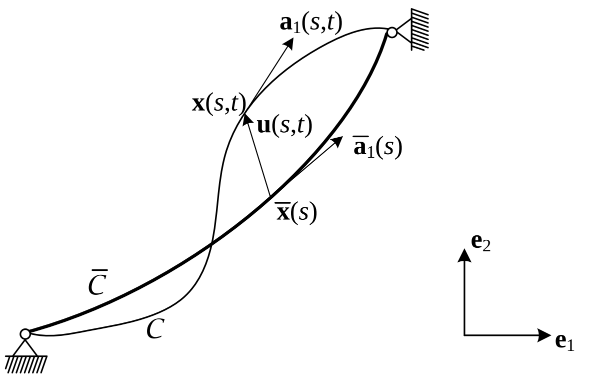

Figure 2. Inclined cable: current C and initial $\bar{C}$ configurations.

An approximate model of shallow cable, prestressed by its own weight and end forces, is used. Shallowness is meant as a small deviation of the cable from its chord. The equations of motion governing the dynamics of an inclined cable, referred to the initial configuration $\bar{C}$, are derived in [2] introducing the following main hypotheses: (a) the cable is planar in the initial configuration, characterized by a small sag-to-length ratio; (b) the Frenet curvature $\bar{\kappa}$ is assumed small and constant along the cable; (c) the prestress $\bar{T}$ is assumed constant along the cable; (d) the $\bar{T}\,/\,EA$ ratio is small which entails that the transverse celerity of the (rectified) cable, $c_t = \sqrt{\bar{T}/m}$, is much smaller than the longitudinal celerity, $c_l = \sqrt{EA/m}$ (EA being the axial stiffness of the cable); (e) the magnitude of the transverse displacements v (in normal direction) and w (in binormal direction, out of the cable plane) is assumed to be of the same order of the sag, whereas the tangential displacement is assumed to be smaller. Actually the hypotheses (b) and (c) are not fulfilled by inclined, long cables. For this reason, reference constant value for the curvature and the prestress, κ^* and T^*, are considered. From a technical point of view, the choice most reasonable is to use values deduced from the point of maximum sag of the cable.

By performing a static condensation of the tangential displacement (i.e., assuming that the strain is constant), the nonlinear transverse motion of the cable is governed by the following, simplified, equations:

$$
\begin{cases}
-m\ddot{v} + (T^* + EA\varepsilon)v'' + EA\varepsilon\kappa^* + b_v = 0 \\
-m\ddot{w} + (T^* + EA\varepsilon)w'' + b_w = 0
\end{cases} \tag{4}
$$

where b_v, b_w contain the wind-induced dynamic external forces, which can be conveniently expressed in the Frenet triad (see, e.g., [11]). The strain ε assumes the following expression:

$$
\varepsilon(t) = -\frac{\kappa^*}{L}\int_0^L v\,\mathrm{d}s + \frac{1}{2L}\int_0^L (v'^2 + w'^2)\,\mathrm{d}s \tag{5}
$$

Then the motion is governed by two integro-differential equations in the transverse displacements $v(s,t)$ and $w(s,t)$. The problem is completed by the geometric boundary conditions.

A discrete multi degree-of-freedom model can be obtained from Equations (4)-(5). The components of motion are assumed as:

$$
\begin{aligned}
v(s,t) &= \sum_{k=1}^{N} q_{vk}(t)\phi_{vk}(s) \\
w(s,t) &= \sum_{k=1}^{N} q_{wk}(t)\phi_{wk}(s)
\end{aligned} \tag{6}
$$

where N is the number of modes considered in the analysis, $q_{ik}(t), i = v, w$ describe the temporal behavior (motion amplitude) and $\phi_{ik}(s)$ are the in-plane (i=v) and out-of-plane (i=w) modes, respectively. Since the cable sag is small, a reasonable approximation is to assume the mode shapes of the taut string (e.g., [12]), i.e. $\phi_{vk}(s) = \phi_{wk}(s) = \sin(k\pi s/L)$. By applying the Galerkin method and adding the contribution of the mechanical damping, a system of $2N$ ordinary differential equations is obtained:

$$
\begin{cases}
M\ddot{q}_{vi}(t) + 2\xi_{vi}\omega_{vi}M\dot{q}_{vi}(t) + M\,\omega_{vi}^2 q_{vi}(t) + \\
-\sum_{k=1}^{N}\frac{EA\kappa^*\pi i^2}{2L}\frac{1+(-1)^{1+k}}{k}q_{vk}(t)q_{vi}(t) + \\
-\sum_{k=1}^{N}\frac{EA\kappa^*\pi k^2}{4L}\frac{1+(-1)^{1+i}}{i}\left[q_{vk}^2(t)+q_{wk}^2(t)\right] + \\
+\sum_{k=1}^{N}\frac{EA\pi^4 k^2 i^2}{8L^3}\left[q_{vk}^2(t)+q_{wk}^2(t)\right]q_{vi}(t) + = F_{vi}(t) \\
\\
M\ddot{q}_{wi}(t) + 2\xi_{wi}\omega_{wi}M\dot{q}_{wi}(t) + M\,\omega_{wi}^2 q_{wi}(t) + \\
-\sum_{k=1}^{N}\frac{EA\kappa^*\pi i^2}{2L}\frac{1+(-1)^{1+k}}{k}q_{vk}(t)q_{wi}(t) + \\
+\sum_{k=1}^{N}\frac{EA\pi^4 k^2 i^2}{8L^3}\left[q_{vk}^2(t)+q_{wk}^2(t)\right]q_{wi}(t) + = F_{wi}(t)
\end{cases} \tag{7}
$$

where i=1,…,N, $M = mL/2$, ξ_{vi} and ξ_{wi} are the in-plane and out-of-plane mechanical damping ratios, ω_{vi} and ω_{wi} are the in-plane and out-of-plane natural circular frequencies of the cable, F_{vi} and F_{wi} are the modal forces including the incremental wind loads. Despite the mode shapes considered are those of the taut string for both directions, the correct in-plane natural frequencies ω_{vi} can be approximated by the expressions reported in [12].

The expression of modal forces in Equations (7) is:

$$F_{vi}(t) = \int_0^L b_v(s,t)\phi_{vi}(s)\,\mathrm{d}s \tag{8}$$

$$F_{wi}(t) = \int_0^L b_w(s,t)\phi_{wi}(s)\,\mathrm{d}s \tag{9}$$

Details on the characterization of the wind-induced dynamic forces will be discussed in the next Section.

Aerodynamic loading on stay cables

Taking into consideration only the longitudinal turbulence in the first beat, the wind field representation was made via a series expansion which leads to advantages in the numerical simulation. In this way the wind-induced modal forces can be immediately deduced.

In order to complete the discretization of the equations of motion, it is necessary to discretize the longitudinal wind-turbulence field $u'(s,t)$. In analogy to the structural modeling, the representation of $u'(s,t)$ is carried out via the Galerkin method [9]:

$$u'(s,t) = \sum_{k=1}^{Nw} y_k(t)\theta_k(s) \tag{10}$$

$\theta_k(s)$ being the spatial representation of turbulence, $y_k(t)$ its amplitude variation in time, N_w the number of modes employed to represent the wind turbulence field. In order to limit the computational burden, at the same time ensuring a good description of the forces on the cable, we assume to consider the same number of modes used in the mechanical description, i.e. N_w=N. For the case analyzed, in which the cable mode shapes are represented by sinusoidal functions, a convenient orthonormal basis is:

$$\theta_k(s) = \sqrt{\frac{2}{L}}\sin\left(k\pi\frac{s}{L}\right) \tag{11}$$

The power spectral density matrix of the driving process $\mathbf{y}(t)$ referring to two generic cable stays, namely 1 and 2, is obtained through the expression:

$$S_{y_h y_k}(\omega) = \int_0^{L_1}\int_0^{L_2} \theta_h(s_1)\theta_k(s_1)S_{v1v2}\left(\mathbf{x}_w(s_1),\mathbf{x}_w(s_2);\omega\right)\mathrm{d}s_1\,\mathrm{d}s_2 \tag{12}$$

where the cross-power spectral density function is known through the coordinates of the cable points with respect to the wind reference system (Figure 3); it is expressed in terms of auto-spectra and coherence functions in the form:

$$S_{v1v2}\left(\mathbf{x}_w(s_1),\mathbf{x}_w(s_2);\omega\right) = \sqrt{S_v\left(\mathbf{x}_w(s_1);\omega\right)S_v\left(\mathbf{x}_w(s_2);\omega\right)}\cdot \\ \cdot\mathrm{Coh}\left(\mathbf{x}_w(s_1),\mathbf{x}_w(s_2);\omega\right) \tag{13}$$

From [14] the auto-spectrum of the longitudinal turbulence component is a function of the z_w coordinate only and it is given in the form:

$$S_v\left(z_w;\omega\right) = \frac{\sigma_u^2 d_u L_u / U(z_w)}{\left[1+1.5d_u\omega L_u/\left(2\pi U(z_w)\right)\right]^{5/3}} \tag{14}$$

where d_u=6.868, $U(z_w)$ is the mean wind velocity at the height z_w, L_u is the integral length scale of the longitudinal turbulence component, σ_u^2 being its variance [14].

The two-point coherence tends to decrease on increasing separation distance and frequency. The representation of the general case in which 2 points are not aligned with the wind reference system can be described by (e.g., [14]):

$$\mathrm{Coh}\left(\mathbf{x}_{w1},\mathbf{x}_{w2};\omega\right) = \exp\left\{-\frac{\omega\sum_{r=x,y,z} C_{ru}\left|r_{w1}-r_{w2}\right|}{\pi\left[U(z_{w1})+U(z_{w2})\right]}\right\} \tag{15}$$

where C_{ru} are the exponential decay coefficient of the longitudinal turbulence component (e.g., C_{xu}=3, C_{yu}=10, C_{zu}=10, from [14]).

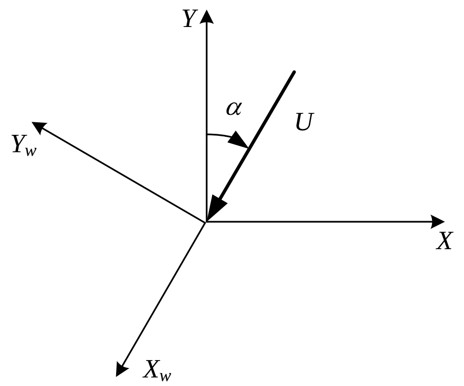

Figure 3. Planar representation of the global (X,Y) and wind (X_w,Y_w) reference systems; the direction of the mean wind speed is indicated by the angle α.

The digital simulation of the turbulence amplitude $y_k(t)$ is finally obtained implementing the matrix $S_{y_h y_k}(\omega)$, h,k=1,…,N_{tot} through the simulation procedure described in [3], N_{tot} being the total number of modes considered for the group of stay cables considered. In this way the wind correlation is automatically taken into account for all the cables included in the group. The simulation is performed through an FFT-based algorithm; $N_T = 2^{15}$ points (corresponding to 6553.6 seconds assuming, for instance, a sampling period equal to 0.2 seconds) are considered. As an example, 3600 sec of simulated time-histories for a group consisting of 50 cables are easily obtained.

Work in progress

The most burdensome step in the simulation is the calculation of the double integer (12) for the peculiarity of the functions contained therein. A semi-analytical method is being developed that allows its evaluation to be extremely cost effective. Moreover, by using a quasi-steady modelling of aeroelastic forces for elements having an arbitrary attitude in the wind field (e.g., [15]), a 3D wind field can be obtained and used in Eq. (9) expressing modal forces.

Following these developments, Equations (7) represent the 2×N nonlinear equations of motion of the generic cable subjected to buffeting, belonging to a group of stays. They can be numerically integrated in the time domain through a standard Runge-Kutta algorithm. The mechanical damping ratios have been set to 0.5% for both directions and all the modes considered. To eliminate the possible transient effects on the response, a half cosine taper window is applied to the initial part of data; vibrations at the end of this time have been chosen as initial conditions for the integration which has to provide the cable dynamic response for the time considered (for instance, 3600 seconds) of wind-induced excitation. Once the 2×N amplitudes q_{vk}(t) and q_{wk}(t), k=1,..N, are obtained, the dynamic displacements referred to the static initial configuration are achieved by the modal expansion, Eq. (6).

Of particular interest is the investigation of the actual role of the mechanical nonlinearities and the possible non-Gaussian behavior of the stay cable response.

References

[1] Irvine, H.M., *Cable Structures*, MIT Press, Cambridge, MA, 1981.

[2] Luongo, A. & Zulli, D., *Mathematical Models of Beams and Cables*, ISTE-Wiley, 2013.

[3] Carassale, L. & Solari, G., Monte Carlo simulation of wind velocity fields on complex structures, *Journal of Wind Engineering and Industrial Aerodynamics*, **94**, 2006, 323-339.

[4] Bartoli, G., Cluni, F., Gusella, V. & Procino, L., Dynamics of cable under wind action: wind tunnel experimental analysis, *Journal of Wind Engineering and Industrial Aerodynamics*, **94**, 2006, 259–273.

[5] Rega, G., Nonlinear vibrations of suspended cables—part II: deterministic phenomena, *Applied Mechanics Review*, **57** (6), 2004, 479–514.

[6] Ibrahim, R.A., Nonlinear vibrations of suspended cables—part III: random excitation and interaction with fluid flow, *Applied Mechanics Review*, **57** (6), 2004, 515–549.

[7] Chang, W.K. & Ibrahim, R.A., Multiple internal resonance in suspended cables under random in-plane loading, *Nonlinear Dynamics*, **12**, 1997, 275–303.

[8] Impollonia, N., Ricciardi, G. & Saitta, F., Dynamics of shallow cables under turbulent wind: A nonlinear finite element approach, *International Journal of Structural Stability and Dynamics*, **11**(4), 2011, 755-774

[9] Carassale, L. & Piccardo, G., Non-linear discrete models for the stochastic analysis of cables in turbulent wind, *International Journal of Non-Linear Mechanics*, **45**, 2010, 219-231.

[10] Luongo, A. & Zulli, D., Dynamic instability of inclined cables under combined wind flow and support motion, *Nonlinear Dynamics*, **67**, 2012, 71-87.

[11] Luongo, A. & Piccardo, G., Non-linear galloping of sagged cables in 1:2 internal resonance, *Journal of Sound and Vibration*, **214**(5), 1998, 915-940.

[12] Warnitchai, P., Fujino, Y. & Susumpow, T., A non-linear dynamic model for cables and its application to a cable-structure system, *Journal of Sound and Vibration*, **187**(4), 1995, 695-712.

[13] Carassale, L., Freda, A. & Piccardo, G., Aeroelastic forces on yawed circular cylinders: Quasi-steady modeling and aerodynamic instability, *Wind and Structures*, **8**(5), 2005, 373-388.

[14] Solari, G. & Piccardo, G., Probabilistic 3D turbulence modeling for gust buffeting of structures, *Probabilistic Engineering Mechanics*, **16**, 2001, 73-86.