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Flutter Analysis of Long Span Suspension Bridge Considering Aerostatic Torsional Angles

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Abstract

To consider the effect of aerostatic torsional angles, a multi-mode flutter analysis method of a long span bridge is presented which combined with the aerostatic analysis. The detailed flutter analysis of a suspension bridge with a flat box girder was carried out by using the presented method. The results indicated that the flutter onset speed of 1660m main-span suspension bridge could decrease dramatically while considering the effect of aerostatic torsional angles, especially under 5 degree attack angle. A brief discussion on the mechanism of aerostatic effects on flutter is expounded in the end.

Introduction

Flutter analysis method of long span bridges have taken nonlinearity into consideration [1-4]. The results of flutter and buffeting analysis long span suspension bridge showed that, the influences of wind load on long span bridge structural behaviours are as follow: (1) Static wind loads lead to attack angle adjustment that would change the incidence of airflow which can change self-excited aerodynamic force of girders. (2) The additional attack angles varying along bridge span, leads to differences of aerodynamic force, the three-dimensional effects. (3) The dynamic behaviour of structure changes since the change of structure stiffness due to static wind loads.

Effect of static torsional angle induced by strong wind on flutter instability has been taken into account in recent years, since the increment of attack angle would weaken the performance of aerodynamic stability of long-span bridges [5-7]. The critical flutter speed of Jiangyin Yangtze River Bridge under 3° wind attack angle is 46.6m/s, and the attack angle adjustment caused by aerostatic load is 0.62°. The critical flutter wind speed of Sutong Bridge under 3° wind attack angle is 82.2m/s, and the attack angle adjustment caused by static wind load is 1.5°. Researches show that the longer the bridge span is and the more flexible the structure is, the more obvious aerostatic effects are. Therefore, it is necessary to study the mechanism of aerostatic effects on flutter performance of long span bridges.

Analysis of Flutter Using State-Space Method

The governing equations of motion with respect to the static equilibrium position of a bridge excited by aerodynamic forces with *n* DOFs are given in a matrix form by

$$\mathbf{M}\ddot{\mathbf{\Delta}} + \mathbf{C}\dot{\mathbf{\Delta}} + \mathbf{K}\mathbf{\Delta} = \mathbf{F}_{se} \tag{1}$$

Where **M**, **C**, and **K** = mass, damping and stiffness matrices, respectively; Δ= nodal displacement vector; **F** indicates the nodal force vector; each dot denotes the partial differentiation with respect to time *t*; and the subscripts *se* and *b* represent the self-excited and turbulence-induced buffeting force components, respectively.

Select *m* mode of vibration $\mathbf{\Psi}$ from structure inherent vibration mode, and structural response Δ is expressed as

$$\mathbf{\Delta} = \mathbf{\Psi}\mathbf{q} \tag{2}$$

Where, *q*=generalized coordinates of m vibration mode.

Using the Roger rational function, to express self-excited aerodynamic force ***F*** in complex field:

$$\hat{\mathbf{F}}_{se} = \mathbf{V}_f \tilde{\mathbf{Q}} \hat{\mathbf{q}} \tag{3}$$

$$\mathbf{V}_f = \begin{bmatrix} V_L & & \\ & V_D & \\ & & V_M \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\rho U^2 B & & \\ & \frac{1}{2}\rho U^2 B & \\ & & \frac{1}{2}\rho U^2 B^2 \end{bmatrix} \tag{4}$$

$$\tilde{\mathbf{Q}} = \mathbf{A}_1 + \mathbf{A}_2 p + \sum_{n=1}^{m} \frac{\mathbf{A}_{n+2} p}{p + \lambda_n} \tag{5}$$

$$\hat{\mathbf{q}} = \begin{bmatrix} \hat{h}/B & \hat{p}/B & \hat{\alpha} \end{bmatrix}^T \tag{6}$$

Where, ^ on behalf of Laplace transform, p on behalf of dimensionless Laplace variable.

The above flutter motion can be expressed in the following state-space format:

$$\dot{\bar{\mathbf{Y}}} = \bar{\mathbf{A}}\bar{\mathbf{Y}} \tag{7}$$

$$\bar{\mathbf{Y}} = \left\{ \mathbf{q} \quad \dot{\mathbf{q}} \quad \mathbf{\Psi}^T \mathbf{\Phi}_{se1} \quad \cdots \quad \mathbf{\Psi}^T \mathbf{\Phi}_{sem} \right\}^T \tag{8}$$

$$\bar{\mathbf{A}}=\left\{\begin{array}{ccccc} 0 & \mathbf{I} & 0 & \cdots & 0 \\ -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}} & -\bar{\mathbf{M}}^{-1}\bar{\mathbf{C}} & \frac{1}{2}\rho U^2\bar{\mathbf{M}}^{-1} & \cdots & \frac{1}{2}\rho U^2\bar{\mathbf{M}}^{-1} \\ 0 & \bar{\mathbf{A}}_3 & -\frac{\lambda_1 U}{B}\mathbf{I} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \bar{\mathbf{A}}_{2+m} & 0 & \cdots & -\frac{\lambda_m U}{B}\mathbf{I} \end{array}\right\} \quad (9)$$

$$\bar{\mathbf{M}}=\mathbf{\Psi}^{\mathrm{T}}\mathbf{M}\mathbf{\Psi} \quad (10)$$

$$\bar{\mathbf{C}}=\mathbf{\Psi}^{\mathrm{T}}\mathbf{C}\mathbf{\Psi}-\frac{1}{2}\rho UB\mathbf{\Psi}^{\mathrm{T}}\mathbf{b}^{T}\mathbf{A}_2\mathbf{b}\mathbf{\Psi} \quad (11)$$

$$\bar{\mathbf{K}}=\mathbf{\Psi}^{\mathrm{T}}\mathbf{K}\mathbf{\Psi}-\frac{1}{2}\rho U^2\mathbf{\Psi}^{\mathrm{T}}\mathbf{b}^{T}\mathbf{A}_1\mathbf{b}\mathbf{\Psi} \quad (12)$$

Where

$$\mathbf{b}=\begin{bmatrix} 1 & & \\ & 1 & \\ & & B \end{bmatrix};\mathbf{A}_i=\begin{bmatrix} A_{iLh} & A_{iLp} & A_{iL\alpha} \\ A_{iDh} & A_{iDp} & A_{iD\alpha} \\ A_{iMh} & A_{iMp} & A_{iM\alpha} \end{bmatrix}, i=1\cdots m$$

The above equations are linear, and the aerodynamic parameters in equations are independent on structure vibration frequency. Therefore, the calculation of structure inherent mode eigenvalue and all unsteady aerodynamic mode need not iterative procedure.

Procedure of Multi-mode Flutter Analysis with Static Aerodynamics Effects

Generally, critical flutter speed of a long-span bridge is much higher than the design wind speed. However, the attack angle adjustment under strong wind is not negligible as the flexibility of a long span suspension bridge, and the increment of attack angle would weaken the aerodynamic stability. Therefore, static aerodynamics effects should be taken into account in multi-mode flutter analysis.

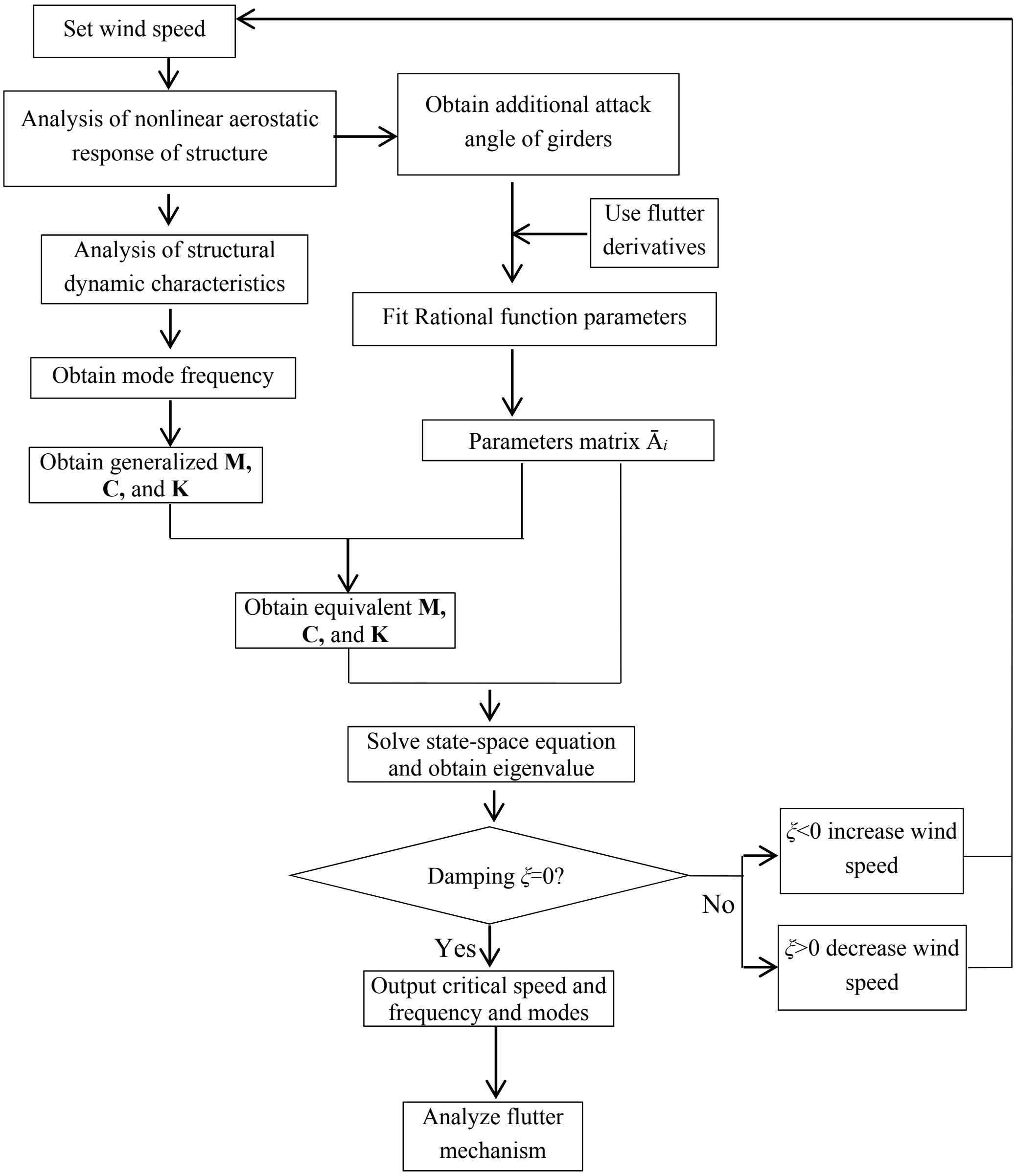

Figure. 1 Flow chart of multi-mode flutter analysis with consideration of aerostatic effects

Multi-mode flutter analysis includes five parts: Structure nonlinear aerostatic response analysis; structure dynamic characteristics analysis; the formation of state-space equation; search of critical flutter speed and flutter modes analysis. Compared with multi-mode flutter analysis without consideration of aerostatic effects, the added part of this analysis method including structural nonlinear aerostatic response analysis and the updated parameters of state-space equation. The difference is attributed to the changes of self-excited force along bridge span. With no consideration of aerostatic effects, the effective attack angles along bridge span are initial attack angles, and the self-excited force will keep the same under different wind speeds. On the condition of flutter analysis with aerostatic effect, the attack angles adjustment along bridge span are different, so the effective attack angles are different, which cause the difference in self-excited aerodynamic force.

The flow chart of multi-mode flutter analysis with consideration of aerostatic effects is shown in figure. 1.

As a result of the small stiffness and damping of a long span bridge, the proportion of lateral movement in wind-induced vibration increases, so the corresponding flutter derivatives should be considered in flutter analysis.

The corresponding flutter derivatives related to lateral motion can be calculated by quasi-steady theory:

$$P_1^* = -2C_D/K\,, P_2^* = \left(C_L - C_D'\right)/\left(2K\right), P_3^* = C_D'/K^2$$

$$P_5^* = \left(C_D' - C_L\right)/K\,, H_5^* = 2C_L/K\,, A_5^* = -4C_M/K \qquad (13)$$

$$P_4^* = P_6^* = H_6^* = A_6^* = 0 \qquad (14)$$

Where C_L, C_D, and C_M= static lift, drag, and moment coefficients, respectively, and C_D'=$C_D/d\alpha$.

Case Study: A Suspension Bridge with 1660m Main Span

Conducted Multi-mode flutter analysis of a twin-box girder suspension bridge with 1660m main span. Compared the flutter calculation results with and without aerostatic effects.

The main natural vibration frequencies and modes of the 1660m main span suspension bridge are showed in table. 1

Table 2 showed the flutter analysis results of a three-span suspension bridge under different work conditions. It can be concluded that, aerostatic effects has limited effects on its flutter performance under 0°. The critical flutter speed would decrease by 5.8% with the consideration of aerostatic effects. Under 3°attack angle, the critical flutter speed would reduce by 11% with the consideration of aerostatic effects. Under 5°, the critical flutter speed would decrease by 14% with the consideration of aerostatic effects. Although the accessional attack angle induced by static wind load more than 1°, the accessional attack angles of the side span girders are negative, leading to the three-dimensional effects of the whole bridge.

n	*f*	Mode shape
1	0.0693	L-S-1
2	0.0892	V-A-1
3	0.1010	V-S-1
4	0.1328	V-S-2
9	0.1684	T-S-1
10	0.1771	V-A-1
13	0.1946	L-A-1
14	0.2008	V-S-3
15	0.2205	L-A-2
16	0.2221	T-A-1
17	0.2309	V-S-4

Table. 1 The mainly natural vibration frequency and mode

Static wind	0º		3º		5º	
	$v/m{\cdot}s^{-1}$	f/Hz	$v/m{\cdot}s^{-1}$	f/Hz	$v/m{\cdot}s^{-1}$	f/Hz
Without Effect	152.4	0.148	130.8	0.1492	121.4	0.151
With Effect	143.6	0.145	116.7	0.1508	104.2	0.152

Table. 2 Flutter analysis results of three-span suspension bridge

Static wind	Attack angle	*n*	1	2	3	9
Without Aerostatic Effect	0º	factor	0.278	0.042	0.485	0.853
		phase/º	7	17	14	33
	3º	factor	0.201	0.028	0.521	0.792
		phase /º	11	15	27	52
	5º	factor	0.186	0.015	0.562	0.757
		phase /º	16	24	38	78
With Aerostatic Effect	0º	factor	0.255	0.033	0.609	0.823
		phase /º	5	8	23	62
	3º	factor	0.228	0.067	0.523	0.765
		phase /º	22	10	30	73
	5º	factor	0.186	0.025	0.489	0.695
		phase /º	10	21	39	88

Table. 3 Participation factors and phases of complex mode in critical flutter state

Table 3 showed the amplitude and phase of each complex mode. It can be seen that the coefficient and phase of bridge complex mode in flutter differ from each other. However, the stable motions of flutter critical state are coupled motion between first symmetric lateral mode (mode 1), first symmetric vertical mode (mode 3) and first symmetric torsional mode (mode 9), the participation of anti-symmetry modes and high order modes are less. Under different work condition, there is certain phase angle between complex modes by the phase shown in tab. So the flutter performance of the bridge also showed that its torsional phase has negative lag behind the other modes.

Table 4 showed the contribution of aerodynamic stiffness coupling and aerodynamic damping coupling between main modes to flow damping. The results showed that under different conditions, the aerodynamic damping coupling of mode 3 and mode 9 has made a great contribution to flutter.

Static wind	Attack angle	Mode coupling	Damping coupling	Stiffness coupling	Static wind	Attack angle	Mode coupling	Damping coupling	Stiffness coupling
Without Effect	0°	3、3	0.0525	0	With Effect	0°	3、3	0.0695	0
		3、9	-0.1681	0.0485			3、9	-0.2105	0.0536
		9、9	0.0671	0			9、9	0.0874	0
	3°	3、3	0.0755	0		3°	3、3	0.0729	0
		3、9	-0.2277	0.0851			3、9	-0.2499	0.0786
		9、9	0.0671	0			9、9	0.0984	0
	5°	3、3	0.0769	0		5°	3、3	0.0707	0
		3、9	-0.2005	0.0661			3、9	-0.2188	0.0663
		9、9	0.0575	0			9、9	0.0818	0

Table. 4 The aerodynamic damping of three span continuous suspension bridge at the critical flutter status

Conclusion

Conducted detailed multi-mode flutter analysis of a suspension bridge with consideration of static aerodynamics effect on torsional angle, and conclusions are mainly as follows.

(1) The nonlinear flutter analysis method has been established which can take the aerostatic effect into account via Roger rational function.

(2) The critical flutter speed of 1660m suspension bridge considering attack angle adjustment is lower than the critical speed without the consideration of static aerodynamics effect.

(3) The stable motions of flutter of 1660m suspension bridge are mainly coupled with first symmetric bending mode (mode 3) and first symmetric torsional mode (mode 9) and its torsional phase lag is negative.

(4) The aerodynamic damping of vertical bending mode and torsional mode have made great contribution to flutter instability, and the negative work of self-excited lift force induced by torsional motion plays an important role.

References

[1] Chen, X., Kareem, A & Matsumoto, M., Time Domain Flutter and Buffeting Response Analysis of Bridges, *Journal of Engineering Mechanics.*,126(1), 2000, 7-16.

[2] Chen, X., Kareem, A & Matsumoto, M., Aerodynamic Coupling Effects on Flutter and Buffeting of Bridges, *Journal of Engineering Mechanics.*,126(1), 2000, 17-26.

[3] Chen, X., Kareem, A & Matsumoto, M., Multimode Coupled Flutter and Buffeting Analysis of Long Span Bridges, *Journal of Wind Engineering and Industrial Aerodynamics.*,89, 2001, 649-664.

[4] Chen, X., Improved Understanding of Bimodal Coupled Bridge Flutter Based on Closed-Form Solutions, *Journal of Structural Engineering.*,133(1), 2007, 22-31.

[5] Xiong Long., Liao Haili., & Ma Cunming., Study on aerostatic effects on flutter of kilometre level suspension bridge, *J. Huazhong Univ. of Sci. & Tech. (Natural Science Edition).*,44(12), 2016, 44-49.(in Chinese)

[6] Zhu Le-Dong, Zhu Qing, Guo Zhen-shan. Effect of wind-induced static torsional angle on flutter performance of bridge via section model test[J]. Journal of vibration and shock,2011, 23(5):23-26. (in Chinese)

[7] Ouyang Ke-jian, Chen Zheng-qing. Influence of static wind addition attack angle on flutter performance of bridges[J]. Journal of vibration and shock,2015, 34(2): 45-49.