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A Semi-analytical Solution for Three-dimensional Aerodynamic Admittance of Streamlined Bridge Deck in Turbulent Flow

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Abstract

The aim of the present work is to propose a generalized approach to derive the closed-form solution for three-dimensional aerodynamic admittance (3D AAF), which can describe the spatial distribution of the lift force on streamlined bridge decks in turbulent flow. Using Rinber's three-dimensional theory, the solutions presented herein are only dependent on the empirical coherence models corresponding to the lift force and turbulence. Therefore, the problem of 3D AAF became an issue of experimentally determining a more accurate coherence model of the lift force. Therefore, an empirical coherence model is experimentally proposed by introducing the reduced separations associated with the characteristic lengths of body and the length scales of turbulence, taking the effects of three-dimensional turbulence on the spatial correlation of the gust-loading into consideration. It should be noted that the derived results are semi-analytical solutions due to the floating parameters in those coherence models. Notwithstanding, it provides us with an explicit insight into the effects of flow three-dimensionality and allows us quantitatively study the unsteady behavior of the lift force on streamlined bridge decks.

Introduction

Aerodynamic admittance is of great significance in studying the unsteady behaviour of the lift forces on bluff bodies in turbulent flow. The topic is of interest for streamlined bridge decks. The key point of this study is the description of the three-dimensional spectral tensor of the lift in Fourier-space, as well as the derivation of analytical solutions for the 3D AAF that retain most of the information regarding the three-dimensional effects of turbulence.

In practical gust-loading problem, a strip-theory assumption [1] is usually adopted, wherein the spatial correlation of the turbulence is used to describe the three-dimensional properties of the lift force. It is well known that the strip-theory assumption becomes more accurate when as the length-scale of the turbulence increases. However, published experimental studies [4-8] show that the lift force is more correlated than the turbulence when the length scale of the oncoming flow is not large compared with the characteristic length of the body. In modern bridge engineering, the width of the streamlined bridge deck (≈20-50 m) is approaching the length scales of the vertical gusts (≈30-50 m), which requires a more accurate three-dimensional model for the gust-loading. In fact, this three-dimensional problem relies on the exact description of the aerodynamic admittance in the spectral tensor of the lift, since the aerodynamic admittance is generally regarded as the aerodynamic transfer function of a linear time-invariant system.

Originally, the 3D AAF was proposed to study the three-dimensional problem of a finite aerofoil in homogeneous turbulence by taking the spanwise variations into account [9], which was derived from thin-wing theory and represented essentially a generalization of Sears' results to wings of finite span. A more general formalism of the problem for application to three-dimensionally varying turbulence was suggested by Ribner [11]. The mean-square lift caused by a homogeneous turbulent field can be represented as a superposition of plane sinusoidal waves of all orientations and wavelengths:

$$\left\langle L^2 \right\rangle = \iiint_{-\infty}^{\infty} \left| \chi\left(\vec{\mathrm{k}}\right) \right|^2 S_w\left(\vec{\mathrm{k}}\right) dk_1 dk_2 dk_3 \tag{1}$$

where $\vec{\mathrm{k}}$ is the wavenumber vector having components k_1, k_2, k_3 in the *x*, *y* and *z* directions respectively. For a surface of infinitesimal thickness, $\left|\chi\left(\vec{\mathrm{k}}\right)\right|^2$ reduces to

$$\left|\chi\left(\vec{\mathrm{k}}\right)\right|^2 = \left|\chi\left(k_1, k_2\right)\right|^2 \tag{2}$$

Inspired by Ribner's suggestion, Graham [3] numerically calculated the exact two-wavenumber aerodynamic admittance (3D AAF) based on lift-surface theory for an aerofoil of infinite span length due to the gusts with arbitrary horizontal wavevectors (k_1, k_2), which can describe the three-dimensionality of lift force. Independently, Filotas [2] derived an approximate close-formed expression for the two-wavenumber aerodynamic transfer function of a aerofoil passing through an inclined sinusoidal gust, which was accurate in the high-frequency limit or very low-frequency limit.

With respect to the lift on streamlined bridge decks, the potential theory is certainly not an option due to the complicated separation of flow. The problem of the 3D AAF is a classic one which has attracted considerable attention. In previous studies, it was found that the 3D AAF of streamlined bridge decks was influenced by the characteristic length of body and the integral length scales of turbulence. Thus, the empirical admittance model was experimentally obtained using the mathematical regression method [6]. The goal of the empirical model focus on predicting the buffeting response instead of revealing the effects of flow three-dimensionality on the spatial distribution of the unsteady lift force. In the preceding paper [7], we developed a general approach for numerically obtaining the two-wavenumber spectrum and admittance of the lift on thin aerofoil in grid-generated turbulence, which is highly accurate in comparison with Sears' and Graham's theoretical results. It should be noted that the above approach can be applied to bluff bodies with complicated cross-sectional shapes. In the present paper, we will extend this work from a numerical solution to an analytical solution.

The aim of the present work is to propose a generalized approach based on Ribner's three-dimensional theory, yielding the close-form expressions for the 3D AAF of lift on streamlined bridge decks in turbulent flow. This approach consists of two steps. The first step focuses on experimentally parametrizing the coherence of the lift by taking full account of the spanwise variations of turbulence. The second is to present the close-form expressions of lift admittances using the proposed coherence model, following which the three-dimensional effects of the turbulent flow can be studied by means of the second-order spectral tensor method.

Mathematical model

Before discussing the closed-form solution for the 3D AAF of lift in turbulent flow, we introduce the three-dimensional representation of the lift force on bluff bodies in Fourier-space developed in Li et al. [7]

$$S_L(k_1,k_2)=(\rho Ub)^2\left|\chi_L(k_1,k_2)\right|^2\Big[4C_L^2S_u(k_1,k_2) \\ +\left(C_L^{'}+C_D\right)^2 S_w(k_1,k_2)\Big] \tag{3}$$

with

$$\left.\begin{aligned} S_L(k_1,k_2)&=\Phi_L(k_1,k_2)S_L(k_1)\\ S_u(k_1,k_2)&=\Phi_u(k_1,k_2)S_u(k_1)\\ S_w(k_1,k_2)&=\Phi_w(k_1,k_2)S_w(k_1)\end{aligned}\right\} \tag{4}$$

where ρ, U, b are respectively the air density, mean wind velocity, semi-chord length of the body, C_L, C_D and $C_L^{'}$ are the lift, drag coefficients and lift slope respectively, and $S_u(k_1,k_2)$, $S_w(k_1,k_2)$ are the two-wavenumber spectra of longitudinal and vertical velocity fluctuations respectively, $S_L(k_1)$, $S_u(k_1)$ and $S_w(k_1)$ correspond to the one-point spectra of lift, longitudinal and vertical velocity fluctuations respectively, $\Phi_L(k_1,k_2)$, $\Phi_u(k_1,k_2)$ and $\Phi_u(k_1,k_2)$ are respectively the two-wavenumber coherence of lift, longitudinal and vertical velocity fluctuations.

The one-wavenumber spectrum of lift force at arbitrary strip along the span of the bridge deck can be obtained from two-wavenumber spectrum by integrating out the variable k_2, thus

$$S_L(k_1)=\int_{-\infty}^{\infty}S_L(k_1,k_2)dk_2 \tag{5}$$

The one-point spectra of the turbulence can be obtained similar to equation (5). With respect to streamlined bridge decks, we know that the lift coefficient is negligibly small when the attack angle is zero, which indicates that it is reasonable to neglect the contribution of *u* component of wind fluctuations. Thus, equation (3) reduces to

$$S_L(k_1,k_2)=(\rho Ub)^2\left|\chi_L(k_1,k_2)\right|^2\left(C_L^{'}+C_D\right)^2S_w(k_1)\Phi_w(k_1,k_2) \tag{6}$$

Substitution of equation (6) into (5) yields the one-point spectrum of lift force, we have

$$S_L(k_1)=(\rho Ub)^2\left|\chi_L(k_1)\right|^2\left(C_L^{'}+C_D\right)^2S_w(k_1) \tag{7}$$

where the generalized one-wavenumber AAF is defined by

$$\left|\chi_L(k_1)\right|^2=\int_{-\infty}^{\infty}\left|\chi_L(k_1,k_2)\right|^2\Phi_w(k_1,k_2)dk_2 \tag{8}$$

Now, we turn to the key part of the theoretical approach in the present work, i.e. an equilibrium equation that associates the 3D AAF with the empirical coherence models corresponding to lift and vertical wind fluctuations respectively. Substituting equations (7) and (8) into equation (4), the equilibrium equation is given by:

$$\frac{\left|\chi_L(k_1,k_2)\right|^2}{\left|\chi_L(k_1)\right|^2}=\frac{\Phi_L(k_1,k_2)}{\Phi_w(k_1,k_2)} \tag{9}$$

Inspired by Mugridge's 3D AAF model of a thin aerofoil, we assume that the 3D AAF of streamlined bridge decks can still be expressed in terms of the two-dimensional admittance in transversely fully correlated gusts (2D AAF) and corresponding correction factor as follows:

$$\left|\chi_L(k_1,k_2)\right|^2=\left|\chi_L(k_1,0)\right|^2\left|F_L(k_1,k_2)\right|^2 \tag{10}$$

where $\left|F_L(k_1,k_2)\right|^2$ is the correction factor to the 2D AAF. This assumption is reasonable since the 2D AAF are only dependent on the reduced frequency similar to Sears' function [8]. Of particular note is the fact that at $k_2=0$, equation (10) reduces to 2D AAF, which suggests that the correction factor $\left|F_L(k_1,k_2)\right|^2\equiv 1$ at $k_2=0$. Similarly, $\left|\chi_L(k_1)\right|^2$ can also be expressed as

$$\left|\chi_L(k_1)\right|^2=\left|\chi_L(k_1,0)\right|^2\left|F_L(k_1)\right|^2 \tag{11}$$

where $\left|F_L(k_1)\right|^2$ is the correction factor in generalized one-wavenumber AAF. Comparing equations (10) and (11), the following relation can be obtained

$$\left|F_L(k_1,k_2)\right|^2\Phi_w(k_1,k_2)=\left|F_L(k_1)\right|^2\Phi_L(k_1,k_2) \tag{12}$$

If $k_2=0$, the correlation factor $\left|F_L(k_1)\right|^2$ can be derived as

$$\left|F_L(k_1)\right|^2=\frac{\Phi_w(k_1,0)}{\Phi_L(k_1,0)} \tag{13}$$

Therefore, it is obvious that the 3D AAF only depends on the empirical coherence models related to lift and turbulence.

Empirical coherence model

To investigate the effects of large-scale turbulence on the spanwise correlation of lift acting on streamlined bridge decks, the wind tunnel tests were carried out in an extreme large close-circuit-type wind tunnel with a 22.5 m (width) 4.5 m (height) working section (XNJD-3). The maximum wind speed is 17 m/s and the turbulent intensity is less than 1% under smooth oncoming flow. The large-scale turbulence is generated by spire 23 m upstream of the model, 0.50 m wide at the base, 0.07 m at the top and 4.3 m high. The lateral separation between two adjacent spires is 2.8 m. The tests in the wind tunnel were typically conducted at a mean wind speed of 9.6 m/s. The flow field characteristics were measured by a Cobra Probe as shown in figure 1. Two sets of probes were used to measure the spatial correlation of the *u*, *v*, and *w* components of the wind fluctuations simultaneously. The tests were carried out at the position, without the rigid model in place. The sampling frequency was set at 256

Hz for a sampling time 180 s. The characteristics of flow conditions for the spire-generated turbulence are listed in table 1.

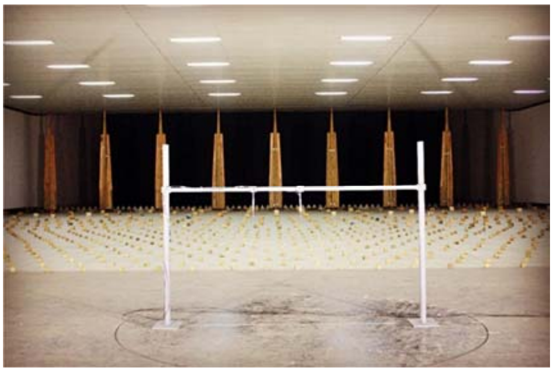

Figure 1. Measurements of the spire-generated turbulence using Cobra Probe.

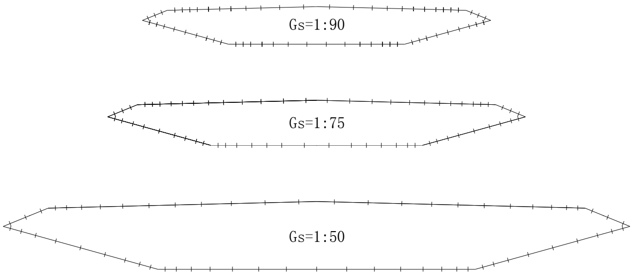

Figure 2. Cross-section of the models investigated with three different geometry scales and the arrangement of the taps.

Integral length scales			turbulence intensity		
L_u (m)	L_v (m)	L_w (m)	I_u (%)	I_v (%)	I_w (%)
0.975	0.348	0.209	11.0	9.2	9.0

Table 1. Characteristics of the spire-generated turbulence.

Sectional models	Gs=50	Gs=75	Gs=90
Width (*B*/m)	0.840	0.560	0.466
Height (*D*/m)	0.090	0.060	0.050

Table 2. Dimensions of the three sectional models with different geometric scales.

To study the effects of characteristic length of the girder and the length scales of gusts on the spatial distribution of buffeting forces, it is necessary to conduct the experiments in different flow fields. In the previous studies, it is found that the ratio of integral scales of turbulence and characteristic lengths of structures is of significance in determining the spanwise correlation of the buffeting forces. However, it is not easy to only change the integral scales of spire-generated turbulence provided that the turbulent intensities are kept constant. To solve this problem, the gust-loading on three sectional models in abovementioned spire-generated turbulence, with the geometric scales of the models (Gs) changed from 50 to 75 and 90, were measured. The cross-sections studied were typical streamlined bridge decks as shown in figure 2. The aspect ratios of the three models (B/D) are keep 9.3 and the characteristic lengths are listed in table 2. The dynamic force measurements were conducted using simultaneous measurements of unsteady surface pressures on six chord-wise strips of the section models. The fluctuating pressures on the sections were measured synchronously by DMS-3400 pressure scanners. The sectional fluctuating forces can be obtained by integrating the pressures over the strip. The wind tunnel tests were typically conducted at a mean wind speed of 9.6 m/s.

In fact, the ratios of B/D and B/L_w^x play a dominant role in determining the spanwise correlation of gust-loading on bluff bodies. Based on an in-depth analysis, a generalized approach was proposed by introducing the reduced separations to take the effects of B/D and B/L_w^x into consideration as follows:

$$\mu_L = \Delta y \zeta_L \tag{14}$$

where

$$\zeta_L = \frac{1}{B}\left(\frac{B}{H}\right)^{0.1}\left(\frac{B}{L_w}\right)^{0.6} \tag{15}$$

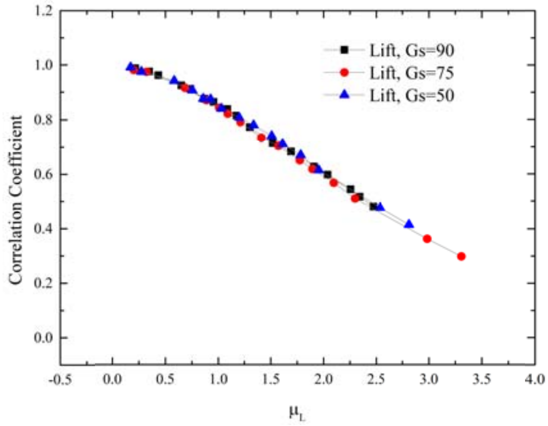

Figure 3. The correlation coefficients of lift on the sectional models varying with reduced separations.

As shown in figure 3, the reduced separation is proved to be of high accuracy and can be applied to improve the present empirical coherence models. In this paper, the double-exponential coherence model [8] is adopted for instance, which is expressed as follows:

$$\mathrm{Coh}_L^{1/2}(k_1,\Delta y)=\frac{\lambda_L \exp(-2\pi \mathrm{A}_L \mu_L)-\mathrm{A}_L \exp(-2\pi \lambda_L \mu_L)}{\lambda_L-\mathrm{A}_L} \tag{16}$$

with

$$\mathrm{A}_L=\left[\sqrt{c_2^2+\left(c_3 k_1 L_w^x\right)^2}\right]^{c_1} \Big/ 2\pi \tag{17}$$

$$\lambda_L=\sqrt{a_1\cdot\left(2\pi b k_1\right)^{a_2}+a_3} \Big/ 2\pi \tag{18}$$

where a_1, a_2, a_3 and c_1, c_2 and c_3 are non-dimensional parameters. The experimental results indicate that this empirical coherence model is of high accuracy, which ensures the reliability of the following solutions for 3D AAF. For simplicity, the coherence of wind fluctuations is described by Jakobsen's model, which is given by

$$\mathrm{Coh}_w^{1/2}(k_1,\Delta y)=\exp\{-\mathrm{A}_w \Delta y\} \tag{19}$$

where

$$\mathrm{A}_w=\left(\sqrt{c_2^2+\left(c_3 L_w^x k_1\right)^2}\right)^{c_1} \tag{20}$$

The two-wavenumber coherence of the above two empirical models are listed as follows

$$\Phi_L=\frac{\lambda_L \mathrm{A}_L \zeta_L^3}{\pi}\cdot\frac{\left(\lambda_L+\mathrm{A}_L\right)}{\left[\left(\lambda_L\zeta_L\right)^2+k_2^2\right]\left[\left(\mathrm{A}_L\zeta_L\right)^2+k_2^2\right]} \tag{21}$$

$$\Phi_w = \frac{2A_w}{\left[A_w^2+\left(2\pi k_2\right)^2\right]} \tag{22}$$

Finally, substitution of equations (21) and (22) into equations (12) and (13) yields the correction factors for 3D AAF, we have

$$F_L\left(k_1\right) = \frac{\zeta_L \lambda_L A_L}{A_w\left(\lambda_L + A_L\right)} \tag{23}$$

$$F_L\left(k_1, k_2\right) = \frac{\zeta_L^4 \lambda_L^2 A_L^2 \left(A_w^2 + k_2^2\right)}{A_w^2\left[\left(\zeta_L \lambda_L\right)^2 + k_2^2\right]\left[\left(\zeta_L A_L\right)^2 + k_2^2\right]} \tag{24}$$

Thus, the semi-analytical solutions for 3D AAF can be obtained, as shown in figure 4. It should be noted that the approach proposed herein is still valid as the empirical model replaced by others, such as Kimura's model.

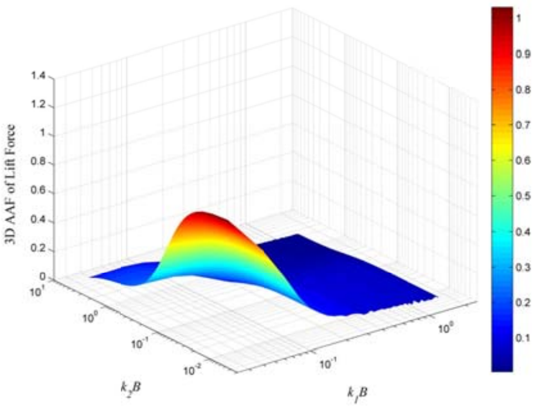

Figure 4. The three-dimensional aerodynamic admittance of lift force on streamlined bridge decks

Conclusions

We have proposed a generalized and efficient approach to obtain the closed-form solutions for the 3D AAF of streamlined bridge decks in Fourier-space. The accuracy of those solutions is only dependent on that of the coherence models corresponding to the lift force and incident fluctuations. Therefore, the problem of 3D AAF became an issue of experimentally determining a more accurate coherence model of lift force.

Using the large-scale turbulent field simulation method, the effects of the characteristic lengths of body and the length scales of turbulence on the spatial correlation of lift force acting on streamlined bridge deck are experimentally studied. The reduced separations, expressed in terms of $\Delta y/B$, B/D and B/L_w^x, are introduced to improve the present empirical coherence models to take account of the effects of flow three-dimensionality.

Based on the proposed coherence model, a closed-form solutions for 3D AAF presented herein are not only of importance in predicting the buffeting response of structures in a more reasonable manner but also mathematically provides us with a more insight into the physical nature of the spatial distribution of the lift force on a bluff body in turbulent flow field. In addition, it should be noted that the proposed approach is also valid as the coherence model replaced by others, which allow us to extend it to line-like structures with complicated cross-sectional shapes.

Acknowledgments

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