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Buffeting predication of long-span bridges using two wavenumber aerodynamic admittance

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Abstract

The conventional gust loading model may not be able to fully describe the buffeting forces of long-span bridges and a large margin error may be induced when the three dimensional (3D) effect of turbulent is significant. In present study, based on 3D theory, a generalized gust loading model for buffeting predication of long-span bridge is established, in which the spectra of buffeting forces are determined by two-wavenumber aerodynamic admittances to deal with the 3D effects of turbulent. The gust loading model shows a dependency on aspect ratio (the ratio of span to the width of a deck). The corresponding buffeting responses can be easily calculated in frequency domain. A numerical example is presented to evaluate the difference of buffeting responses calculated by the proposed and conventional gust loading models. The results show that the flow three-dimensionality cannot be neglected and the conventional gust loading model overestimates the buffeting response over 5% when the aspect-ratio below 5.0, which indicates the invalidation of strip assumption. It is important to deal with 3D effects carefully and essential to use the generalized gust loading model to predict the buffeting response precisely for the long-span bridges with low aspect-ratios.

Introduction

Due to the velocity fluctuations in oncoming flow, buffeting is inevitable for any bridges exposed in natural atmospheric turbulence. Modern long-span bridges tending to be more flexible, lower in inherent damping and lightly in weight, and exhibit an increased susceptibility to wind effects. Thus, it has become greatly important to improve the understanding of structural behavior and precisely predict the buffeting responses of long-span bridges.

It is well known that the aerodynamic admittance is a function to link the wind turbulence and the related fluctuating aerodynamic forces. Based on strip assumption, the one wavenumber aerodynamic admittance is frequently used for computation of buffeting forces of long-span bridge in conventional gust loading models. The most common model of buffeting forces in frequency domain is proposed by Scanlan [1], taking the lift force for example, the spectrum density is expressed as:

$$S_L(\omega) = (\rho U b)^2 \Big[4C_L^2|\chi_{Lu}(\omega)|^2 S_u(\omega) + (C_L' + C_D)^2 |\chi_{Lw}(\omega)|^2 S_w(\omega)\Big] \tag{1}$$

Where ω is the reduced frequency, ρ is the air density, U is the mean wind velocity, b is the semi-width of bridge deck, C_L, C_D are lift and drag coefficients and C_L' is the variations of lift coefficient with angle of wind incidence. S_u and S_w denote power density of the longitudinal u and vertical w components of turbulence. $|\chi_{Lu}(\omega)|^2$, $|\chi_{Lw}(\omega)|^2$ are lift aerodynamic admittance linked to the longitudinal u and vertical w components of turbulence, respectively. In practical buffeting analysis of long-span bridges, the spanwise coherence for lift and turbulence is generally considered to be equal, namely the application of strip assumption.

In fact, due to the distortion of the turbulence approaching the structures, the spanwise coherence of unsteady aerodynamic forces is usually higher than that of turbulence in the cases of gusts in small length scales and structures with low aspect ratios, which has been confirmed theoretically and experimentally [2-5]. Unfortunately, a gust loading model fully considering 3D effects for buffeting analysis of long-span bridge has not been established at present.

In this study, based on the 3D theory, a generalized gust loading model that fully considering the spanwise correlation of aerodynamic forces is established to precisely predict the buffeting response of long-span bridge by frequency domain analyzing method. Furthermore, a comparison of the buffeting response calculated by the conventional and generalized gust loading models are made in a numerical example, the application scope of conventional gust loading model is highlighted.

Motion equations of bridge deck

The coordinate system of bridge deck is shown in figure 1, in which the aerodynamic forces and motions at a bridge section are referred to its central line of gravity in spanwise direction. The buffeting response of a bridge deck in vertical, lateral and torsional directions $h(y,t)$, $p(y,t)$ and $\alpha(y,t)$ can be expressed in terms of the generalized coordinates $q = \{q_i\}$ as

$$\begin{cases} h(y,t) = \sum_i h_i(y) B q_i(t) \\ p(y,t) = \sum_i p_i(y) B q_i(t) \\ \alpha(y,t) = \sum_i \alpha_i(y) q_i(t) \end{cases} \tag{2}$$

where B is the width of bridge deck, $h_i(y)$, $p_i(y)$ and $\alpha_i(y,t)$ are the ith mode shapes in the vertical, lateral and torsional directions, respectively. The associated time-dependent aerodynamic

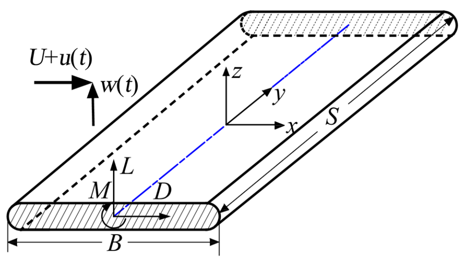

Figure 1: Coordinate system of bridge deck.

force vectors of bridge deck can be separated into self-excited and buffeting contributions. Thus, the governing motion equations of bridge deck in modal coordinates are given by

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{Q_{se}}+\mathbf{Q_b} \tag{3}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the generalized mass, damping ratio and stiffness matrices, respectively; $\mathbf{Q_{se}}$ and $\mathbf{Q_b}$ are the generalized self-excited and buffeting force vectors, respectively, expressed as

$$\mathbf{Q_{se}}=\frac{1}{2}\rho U^2(\mathbf{A_s}\mathbf{q}+\frac{b}{U}\mathbf{A_d}\dot{\mathbf{q}}), \tag{4}$$

$$\mathbf{Q_b}=\frac{1}{2}\rho U^2(\mathbf{A_{bu}}\frac{\mathbf{u}}{U}+\mathbf{A_{bw}}\frac{\mathbf{w}}{U}) \tag{5}$$

where $\mathbf{A_s}$ and $\mathbf{A_d}$ are aerodynamic stiffness and damping matrix, respectively, $\mathbf{A_{bu}}$ and $\mathbf{A_{bw}}$ are buffeting force matrix for u and w fluctuating components, respectively.
The self-excited force components per unit length, i.e. lift, drag and pitching moment are expressed in terms of flutter derivatives $H_i^*,\ P_i^*,\ A_i^*\ (i=1,...,6)$ as

$$L_{se} = \frac{1}{2}\rho U^2 B\left[KH_1^*\frac{\dot{h}}{U}+KH_2^*\frac{B\dot{\alpha}}{U}+KH_3^*\alpha+KH_4^*\frac{h}{B} +KH_5^*\frac{\dot{p}}{U}+KH_6^*\frac{p}{B}\right], \tag{6}$$

$$D_{se} = \frac{1}{2}\rho U^2 B\left[KP_1^*\frac{\dot{p}}{U}+KP_2^*\frac{B\dot{\alpha}}{U}+KP_3^*\alpha+KP_4^*\frac{p}{B} +KP_5^*\frac{\dot{h}}{U}+KP_6^*\frac{h}{B}\right], \tag{7}$$

$$M_{se} = \frac{1}{2}\rho U^2 B^2\left[KA_1^*\frac{\dot{h}}{U}+KA_2^*\frac{B\dot{\alpha}}{U}+KA_3^*\alpha+KA_4^*\frac{h}{B} +KA_5^*\frac{\dot{p}}{U}+KA_6^*\frac{p}{B}\right] \tag{8}$$

where $K=B\omega/U$ and ω is the circular frequency of oscillation. Considering the 3D effects induced by turbulence, the buffeting force components per unit length, i.e. lift, drag and pitching moment are expressed as

$$L_b(\omega,\Delta y) = \rho U b\chi_L(\omega,\Delta y)\left[2C_L u(\omega,\Delta y)+\left(C_L'+C_D\right) w(\omega,\Delta y)\right], \tag{9}$$

$$D_b(\omega,\Delta y) = \rho U b\chi_D(\omega,\Delta y)\left[2C_D u(\omega,\Delta y)+\left(C_D'-C_L\right) w(\omega,\Delta y)\right], \tag{10}$$

$$M_b(\omega,\Delta y) = \rho U b^2\chi_M(\omega,\Delta y)\left[2C_M u(\omega,\Delta y)+C_M' w(\omega,\Delta y)\right] \tag{11}$$

where $\chi_i(\omega,\Delta y)\quad(i=L,D,M)$ is the generalized aerodynamic admittance that considering the influence of longitudinal u and vertical w components of turbulence and taking the spanwise direction variation into account.

Two-wavenumber spectrum of buffeting force

As shown in figure 2, based on 3D theory and assuming the validity of Taylors hypothesis for the motion of the wing through the turbulent field $w(x,y)$, Diederich [6] defined the lift dL on a chordwise strip of width at y as

$$F_L(x,y)=\rho U b C_L'\int\int_{-\infty}^{\infty}\phi(\xi,y-\eta)\,w(x-\xi,\eta)\,d\xi d\eta \tag{12}$$

where $\phi(\xi,y-\eta)$ is the influence function for lift per unit span at y produced by a unit downwash impulse at position (ξ,η) with respect to the wing.

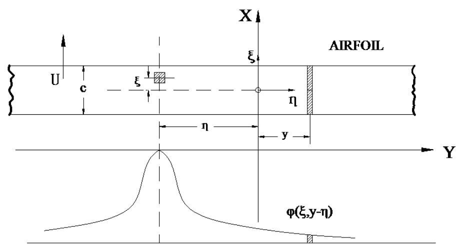

Figure 2: Coordinate systems and sketch of lift influence function. XY system stationary; $\xi\eta$ system follows wing moving with velocity U in X direction. (Li, 2015)

Similarly, for a bridge deck with a span length of S $(S=2s)$, width of B $(B=2b)$ and vertical modal shape $h_i(y)$ $(i=1,2...,n)$, the lift of bridge deck per unit span in ith mode can be written as

$$L_i(x) = \frac{\rho U b}{2s}\int_{-s}^{s}\int\int_{-\infty}^{\infty}h_i(y)\vartheta(\xi,y-\eta)\left[2C_L u(x-\xi,\eta) +(C_L'+C_D)w(x-\xi,\eta)\right]d\xi d\eta dy \tag{13}$$

where $\vartheta(\xi,y-\eta)$ denotes the influence function for lift per unit span at y produced by a unit downwash impulse by longitudinal u and vertical w components at position (ξ,η) with respect to the bridge deck. Therefore, the correlation of the lift for the ith and jth mode can be defined as

$$R_{L_{i,j}}(\Delta x,\Delta y)=\left\langle L_i(x,y)L_j(x+\Delta x,y+\Delta y)\right\rangle \tag{14}$$

Substituting equation (14) into (13) yields the correlation function of lift as

$$\begin{aligned} R_{L_{i,j}}(\Delta x,\Delta y) = {} & \left(\rho U b/2s\right)^2\Big\{4{C_L}^2\int\int_{-s}^{+s}\int\int\int\int_{-\infty}^{\infty} \\ & h_i(\lambda_1)h_j(\lambda_2)\vartheta(\xi_1,\lambda_1-\eta_1) \\ & \vartheta(\xi_2,\lambda_2+\Delta y-\eta_2) \\ & \langle u(x-\xi_1,\eta_1)\,u(x+\Delta x-\xi_2,\eta_2)\rangle \\ & d\xi_1 d\xi_2 d\eta_1 d\eta_2 d\lambda_1 d\lambda_2 + \\ & (C_L'+C_D)^2\int\int_{-s}^{+s}\int\int\int\int_{-\infty}^{\infty}h_i(\lambda_1) \\ & h_j(\lambda_2)\vartheta(\xi_1,\lambda_1-\eta_1)\,\vartheta(\xi_2,\lambda_2+\Delta y-\eta_2) \\ & \langle w(x-\xi_1,\eta_1)\,w(x+\Delta x-\xi_2,\eta_2)\rangle \\ & d\xi_1 d\xi_2 d\eta_1 d\eta_2 d\lambda_1 d\lambda_2\Big\} \end{aligned} \tag{15}$$

In order to simplify the mathematical derivation, only the vertical w component of turbulence is considered. Thus, equation (15) reduced to

$$\begin{aligned} R_{L_{i,j}}(\Delta x,\Delta y) = {} & \left[\rho U b\ (C_L'+C_D)/2s\right]^2 \\ & \int\int_{-s}^{+s}\int\int\int\int_{-\infty}^{\infty}h_i(\lambda_1)h_j(\lambda_2) \\ & \vartheta(\xi_1,\lambda_1-\eta_1)\,\vartheta(\xi_2,\lambda_2+\Delta y-\eta_2) \\ & \langle w(x-\xi_1,\eta_1)\,w(x+\Delta x-\xi_2,\eta_2)\rangle \\ & d\xi_1 d\xi_2 d\eta_1 d\eta_2 d\lambda_1 d\lambda_2 \end{aligned} \tag{16}$$

If the turbulence field is homogeneous in the XY plane, the ensemble average over the velocity products

$R_w(\Delta x+\xi_1-\xi_2,\eta_2-\eta_1)$ is independent of x and can be expressed in terms of a Fourier transform of the non-dimensional two-wavenumber power spectrum $S_w(k_1,k_2)$ as

$$R_w(\Delta x+\xi_1-\xi_2,\eta_2-\eta_1)=\int\int_{-\infty}^{\infty}S_w(k_1,k_2)\exp\left\{\mathrm{i}\left[\widetilde{k_1}(\Delta x+\xi_1-\xi_2)\;+\widetilde{k_2}(\eta_2-\eta_1)\right]\right\}dk_1dk_2$$
$$=\int\int_{-\infty}^{\infty}S_w(k_1,k_2)\exp\left\{\mathrm{i}\left[(\Delta x+\xi_1-\xi_2)k_1/b+(\eta_2-\eta_1)k_2/b\right]\right\}dk_1dk_2 \quad (17)$$

where $\widetilde{k_1}=k_1/b$, $\widetilde{k_2}=k_2/b$, k_1 and k_2 are non-dimensional chordwise and spanwise wavenumbers respectively. Substituting equation (17) into (16), and changing the order of integration gives

$$R_{L_{i,j}}(\Delta x,\Delta y) = \left[\rho Ub\;(C_L'+C_D)/2s\right]^2\int\int_{-\infty}^{\infty}|\chi_L(k_1,k_2)|^2S_w(k_1,k_2)\exp\{i(k_1\Delta x+k_2\Delta y)\}dk_1dk_2\int\int_{-s}^{+s}h_i(\lambda_1)h_j(\lambda_2)\exp\{-ik_2(\lambda_1-\lambda_2)\}d\lambda_1d\lambda_2 \quad (18)$$

In which, $|\chi_L(k_1,k_2)|^2$ is the magnitude of lift aerodynamic transfer function, which is also defined as the two-wavenumber aerodynamic admittance function

$$\chi_L(k_1,k_2) = \frac{1}{2\pi}\int\int_{-\infty}^{\infty}\vartheta(\xi,\eta)\exp\{2\pi\mathrm{i}\left[\xi k_1/b+\eta k_2/b\right]\}d\xi d\eta \quad (19)$$

By Fourier transform of equation (18), the two-wavenumber power spectrum of lift can be obtained as

$$S_{L_{i,j}}(k_1,k_2) = \left[\rho Ub(C_L'+C_D)/2s\right]^2|\chi_L(k_1,k_2)|^2 S_w(k_1,k_2)\int\int_{-s}^{+s}h_i(\lambda_1)h_j(\lambda_2)\exp\left\{-i(\lambda_1-\lambda_2)k_2/b\right\}d\lambda_1d\lambda_2 \quad (20)$$

Taking the contribution of longitudinal u component of turbulence into account, based on equation (20), the two-wavenumber power spectrum of lift can be rewritten as

$$S_{L_{i,j}}(k_1,k_2) = \left[\rho Ub/2s\right]^2|\chi_L(k_1,k_2)|^2\left[4{C_L}^2S_u(k_1,k_2)+(C_L'+C_D)^2S_w(k_1,k_2)\right]\int\int_{-s}^{+s}h_i(\lambda_1)h_j(\lambda_2)\exp\left\{-i(\lambda_1-\lambda_2)k_2/b\right\}d\lambda_1d\lambda_2 \quad (21)$$

Similarly, the two-wavenumber power spectrum of drag and pitch moment per unit span can be obtained as

$$S_{D_{i,j}}(k_1,k_2) = \left[\rho Ub/2s\right]^2|\chi_D(k_1,k_2)|^2\left[4{C_D}^2S_u(k_1,k_2)+(C_D'-C_L)^2S_w(k_1,k_2)\right]\int\int_{-s}^{+s}h_i(\lambda_1)h_j(\lambda_2)\exp\left\{-i(\lambda_1-\lambda_2)k_2/b\right\}d\lambda_1d\lambda_2 \quad (22)$$

$$S_{M_{i,j}}(k_1,k_2) = \left[\rho Ub/2s\right]^2|\chi_M(k_1,k_2)|^2\left[4{C_M}^2S_u(k_1,k_2)+{C_M'}^2S_w(k_1,k_2)\right]\int\int_{-s}^{+s}h_i(\lambda_1)h_j(\lambda_2)\exp\left\{-i(\lambda_1-\lambda_2)k_2/b\right\}d\lambda_1d\lambda_2 \quad (23)$$

When the mode shape $h_i(y)=h_j(y)=1$ (the motionless case), based on the equation (21), the two-wavenumber power spectrum of lift is

$$S_{L_{i,j}}(k_1,k_2) = \left[\rho Ub/2s\right]^2|\chi(k_1,k_2)|^2\left[4{C_L}^2S_u(k_1,k_2)+(C_L'+C_D)^2S_w(k_1,k_2)\right]\left(\frac{\sin k_2\delta}{k_2\delta}\right)^2 \quad (24)$$

where δ is the aspect-ratio.

For the symmetric modes of a simply supported girder, referenced to $y=0$ at the middle span, the heaving mode shape $h_i(y)=h_j(y)=\cos(n\pi y/2s)$, $(n=1,3,...\infty)$, the two-wavenumber power spectrum of lift becomes

$$S_{L_{i,j}}(k_1,k_2) = \left[\rho Ub/2s\right]^2|\chi(k_1,k_2)|^2\left[4{C_L}^2S_u(k_1,k_2)+(C_L'+C_D)^2S_w(k_1,k_2)\right]\left[\frac{2\pi n\cos(k_2\delta)}{(n\pi)^2-(2k_2\delta)^2}\right]^2 \quad (25)$$

For the asymmetric heaving modes of a simply supported girder, submitting the mode shape $h_i(y)=h_j(y)=\sin(n\pi y/2s)$, $(n=1,3,...\infty)$ into equation (21), the two-wavenumber power spectrum of lift can be obtained as

$$S_{L_{i,j}}(k_1,k_2) = \left[\rho Ub/2s\right]^2|\chi(k_1,k_2)|^2\left[4{C_L}^2S_u(k_1,k_2)+(C_L'+C_D)^2S_w(k_1,k_2)\right]\left[\frac{2\pi n\sin(k_2\delta)}{(n\pi)^2-(2k_2\delta)^2}\right]^2 \quad (26)$$

The single wavenumber spectrum of lift force per unit span is computed by integration of the two-wavenumber spectrum over all negative and positive spanwise wavenumbers

$$S_{L_{i,j}}(k_1)=\int_{-\infty}^{+\infty}S_{L_{i,j}}(k_1,k_2)dk_2 \quad (27)$$

Buffeting response

Based on random vibration theory, the power spectral density (PSD) matrices of the vectors of generalized modal response $\mathbf{q}$ and nodal displacement $\mathbf{X}$ are given by

$$\mathbf{S_q}(k_1)=\mathbf{H}^*(k_1)\mathbf{S_{Qb}}(k_1)\mathbf{H}(k_1)^T \quad (28)$$

$$\mathbf{S_X}(k_1)=\Phi\mathbf{H}^*(k_1)\mathbf{S_{Qb}}(k_1)\mathbf{H}(k_1)^{\mathrm{T}}\Phi^T \quad (29)$$

where the subscripts $*$ and T denote the complex conjugate and transpose, respectively, Φ is mode shape matrix, and $\mathbf{H}(k_1)$ is transfer function matrix expressed as

$$\mathbf{H}(k_1) = \left[-(k_1U)^2\mathbf{M}+ik_1U(\mathbf{C}-\rho UB\mathbf{A_d}/2)+\mathbf{K}-\rho U^2\mathbf{A}_s\big/2\right]^{-1} \quad (30)$$

The components of the $\mathbf{S_q}$ and $\mathbf{S_X}$ matrices can be expressed as

$$S_{q_{ij}}(k_1)=\sum_{k=1}^{N}\sum_{l=1}^{N}{H_{ik}}^*(k_1)S_{Qb_{kl}}(k_1)H_{jl}(k_1) \quad (31)$$

$$S_{X_{ij}}(k_1)=\sum_{k=1}^{N}\sum_{l=1}^{N}\Phi_{ik}S_{q_{kl}}(k_1)\Phi_{il} \quad (32)$$

When the cross-spectrum density (CSD) between different generalized buffeting forces are negligible in comparison with PSD, the components of the $\mathbf{S_q}$ and $\mathbf{S_X}$ matrices can be given as

$$S_{q_{ij}}(k_1) = \sum_{k=1}^{N} H_{ik}{}^*(k_1) S_{Qb_{kk}}(k_1) H_{jk}(k_1) \tag{33}$$

The standard deviations of generalized modal response and nodal displacement are given by

$$\sigma_{q_{ij}} = \int_0^{+\infty} S_{q_{ij}}(k_1)dk_1, \qquad \sigma_{X_i} = \int_0^{+\infty} S_{X_i}(k_1)dk_1 \tag{34}$$

Numerical example and analysis

For the sake of illustrating the proposed methodology, the vertical buffeting response of a simply supported bridge with span of 300 m and variable aspect-ratio under vertical wind fluctuation is chosen as an example. The bridge deck used in this study is a thin airfoil section with static coefficients C_L=0.128 and C_L'= 5.558. The fundamental frequency and structural damping ratio of the bridge are 0.179 Hz and 0.01. The height of bridge deck and surface roughness are z=60 m, z_0=0.03 m, respectively. The Lumley-Panofsky Spectrum and exponential coherence function with a decay factor 8 are used. Other parameters are U=40 m/s, m=20 000 Kg/m. The Lumley-Panofsky Spectrum is expressed in terms of wave-number as

$$S_w(k_1) = \frac{3.36u_*^2 z}{2\pi\left[1+10\left(k_1 z/2\pi\right)^{5/3}\right]} \tag{35}$$

where $u_* = KU/\ln\left(z/z_0\right), \quad K \approx 0.4$.
To evaluate the 3D effects of turbulent, the buffeting response of the bridge deck are calculated based on the generalized and conventional gust loading models respectively. In the former calculation, the aerodynamic admittance function (AAF) is used by Blake's [7] approximate expression in two-wavenumber:

$$|\chi(k_1,k_2)|^2 \approx \frac{1}{1+2\pi k_1}\left[\frac{1+3.2(2k_1)^{0.5}}{1+2.4(2k_2)^2+3.2(2k_1)^{0.5}}\right] \tag{36}$$

In the latter calculation the AAF is used as Sears function approximated by Liepmann [8] in single wavenumber:

$$|\chi(k_1)|^2 \approx \frac{1}{1+2\pi k_1} \tag{37}$$

The self-excited forces are omitted and only the fundamental mode is considered for brevity. The ratio of RMS vertical buffeting response at central of main span calculated by two gust loading models is shown in figure 3.
It can be seen that a large margin error may be induced when the bridge has a relatively small aspect-ratios. The conventional gust loading model may overestimate the buffeting response over 5% when the aspect-ratio below 5.0. With the aspect-ratio increased, the response ratio is slowly approach to 1.0. When ($\delta > 10$), the margin error calculated by conventional gust loading model is less than 2% which means the validation of strip assumption for large enough aspect ratio.

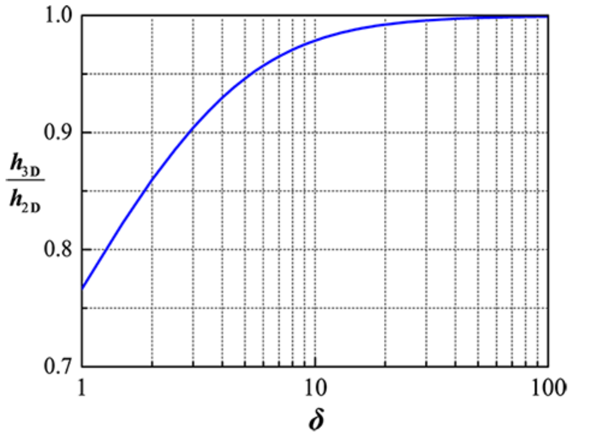

Figure 3: Ratio of RMS vertical buffeting response at central of main span calculated by two gust loading models

Conclusions

A generalized gust loading model that fully considers the 3D effects of turbulence is established to precisely predict the buffeting response of long-span bridges. The proposed gust loading model shows that the aspect-ratio plays an important role in evaluating 3D effects. The numerical example indicates that for the bridge with large aspect ratios ($\delta > 10$), the margin error calculated by conventional gust loading model is less than 2%, which validates the strip assumption. However, the conventional gust loading model overestimates the buffeting results over 5% when the aspect-ratio below 5.0, which is very common for the construction stage for long span bridges and in these cases, the strip assumption is invalidated. Therefore, it is essential to carefully deal with 3D effects and use the generalized gust loading model to calculate the buffeting response for the long-span bridges with low aspect-ratios.

Acknowledgements

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