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A New Quasi-Steady Model for Magnus Effect of Rectangular Plate Windborne Debris

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Abstract

Windborne debris is a major cause of damage during severe storms. In order to understand the potential damage, it is important to model their trajectories correctly. Magnus effect, which is cause by the rotation of windborne debris, has a significant effect on trajectory. Previous quasi-steady models of Magnus effect have a good performance on the computation of square plate trajectory in uniform flow, while their numerical results did not exactly match the experiment data for the rectangular plate. This paper proposes a new quasi-steady model of Magnus effect for rectangular plate in uniform flow. Based on the comparison with existing experiment data available, results show that the new model proposed here provides a better performance on the prediction of trajectories. It is suggested that the fluctuating component of rotational lift and drag should be modeled as a function of both angle of attack (α) and dimensionless spin parameter of the plate (S/S_0), ignoring the change of amplitude of rotational lift and drag force versus dimensionless spin parameter of the plate (S/S_0) may contribute to the distortion of the numerical trajectories.

Keywords: Windborne debris; Trajectories; Quasi-steady model; Autorotation; Rectangular plate.

Introduction

Previous studies have revealed that windborne debris is a major source of building environment damage during severe storms (Karrem [1], Lee [2], Minor [3]). What is more, flying debris will penetrate the building and cause a great damage to the internal property. Wills *et al*. [4] provided a useful classification which divided the debris into three types: compact type, plate-like type and rod like type. According to the NAHB Research Center [5], plate-like debris is the main type of debris which causes the damage of building envelopes in these three types of wind-borne debris. Additionally, plate-like debris has a most complex interaction with the airflow during flying, so the model of plate-like debris trajectory is the most difficult.

In order to assess the potential hazards of windborne debris, the most important part is to model the trajectories of debris correctly. Plate-like type debris has been investigated by a number of researchers. The pioneering work on plate-like was made by Tachikawa [6], who made several great contributions including proposing the motion equation, assuming the aerodynamic as a sum of static component and rotational component and so on. Holmes *et al*. [7] presented a model of the force coefficients to solve the two-dimensional (2D) equation of motion by the quasi-steady model. Beker *et al*. [8] derived a generalised dimensionless and a simplified forms of equations for compact and sheet debris, the large time asymptotic solutions were also derived in their paper for velocities and energies. Richards *et al*. [9] presented a three dimensional calculation model of debris. Kordi *et al*. [10] made an evaluation of quasi-steady theory applied to windborne flat square plate in uniform flow. However, these researchers mainly focused on the square plate, and few work have done on the rectangular plate except Tachikawa [6] and Richards *et al*. [9], while their numerical results did not exactly match the experiment data for the rectangular plate. The offset between the calculation results and experiment data may contribute to the inaccurate model of the rotational lift and drag.

Based on the experiment data provided in Tachikawa [6], a new model of rotational lift and drag of rectangular plate which has a side length ratio of 2 is proposed in this paper. The accuracy of this model is calibrated and validated via the comparison of the simulated data and the wind tunnel data in Tachikawa [6].

Quasi-Steady Model

Equations of Motion

The sketch map of a flate plate moving two-dimensionally in a uniform flow is shown as Figure 1.

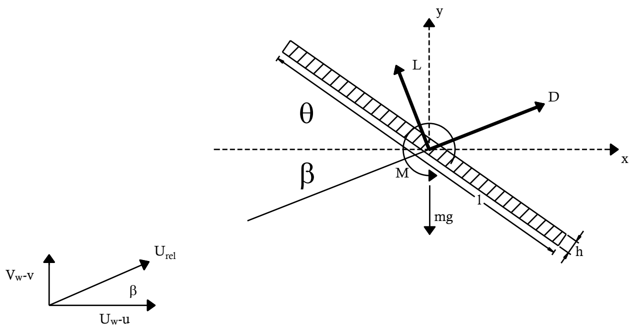


Figure 1. Definition of force and angle

where V_w is the vertical wind velocity; U_w is the horizontal wind velocity; u is the horizontal plate velocity; v is the vertical wind velocity; U_{rel} is relative wind velocity, $U_{rel} = [(U_w - u)^2 + (V_w - v)^2]^{0.5}$; β is the angle of attack of the relative wind respect to the horizontal axis; θ is the angular rotation; h is the plate thickness; l is the plate length which parallel to flow; L is the lift force; D is the drag force; M is the pitching moment;

The equations of motion of a flat plate used in this paper are same as those in Kordi *et al*. [10]. Those equations are shown as below:

$$\frac{d^2x}{dt^2} = (\frac{A\cdot\rho}{2m})[(C_{DS} + C_{DR})(U_w - u) - (C_{LS} + C_{LR})(V_w - v)]U_{rel} \tag{1}$$

$$\frac{d^2y}{dt^2} = (\frac{A\cdot\rho}{2m})[(C_{DS} + C_{DR})(V_w - v) + (C_{LS} + C_{LR})(U_w - u)]U_{rel} - g \tag{2}$$

$$\frac{d^2\theta}{dt^2} = \left(\frac{A\cdot l\cdot\rho}{2I}\right)(C_{MS} + C_{MR})U_{rel}^2 \tag{3}$$

where g is the acceleration of gravity; m is the mass; I is the moment of inertia, $\mathrm{I} = \mathrm{m} \cdot (h^2 + l^2)/12$; ρ is the air density; A is the plate area; x is the horizontal plate displacement; y is the vertical plate displacement; θ is the angular rotation; C_{DS} is the static drag coefficient; C_{DR} is the rotational drag coefficient; C_{LS} is the static lift coefficient; C_{LR} is the rotational lift coefficient; C_{MS} is the static moment coefficient; C_{MR} is the rotational moment coefficient;

Static Force Coefficients

The static drag, lift and pitching moment used in this paper are defined as follows:

$$C_{DS} = C_N \cdot \sin(\alpha) \tag{4}$$

$$C_{LS} = C_N \cdot \cos(\alpha) \tag{5}$$

$$C_{MS} = C_N \cdot (c/l) \tag{6}$$

where C_N is the normal force coefficient; α is the angle of attack of the relative wind respect to the plate, $\alpha = \beta + \theta$; c is the distance of the centre of pressure position from the centre of plate for a rectangular plate. The normal force coefficients and centre of pressure position used herein are those tested by Richards *et al*. [9]. It should be mentioned that in Holmes *et al*. [7] an additional drag coefficient of 0.1 has been added to allow for the skin friction component, while in this paper the calculation result without this additional part has a better agreement with experiment data. Note that the model of *c* fitted by Eq. (9) in Richards *et al*. [9] is a combine of several cases and it is suitable for three dimensional trajectories, while for the case studied in this paper c/l can be expressed in a simpler way which is tentatively assumed as a linear function shown as below:

$$c/l = 0.25 - 0.25/90 \cdot \alpha \tag{7}$$

Quasi-Steady Model for Rotational Force

The drag, lift, and pitch moment components defined in Tachikawa [6] are shown below:

$$C_D = C_{Dr0} \cdot (C_{Dr}/C_{Dr0}) + C_{Ds} - \overline{C_{Ds}} \tag{8}$$

$$C_L = C_{Lr0} \cdot (C_{Lr}/C_{Lr0}) + C_{Ls} \tag{9}$$

$$C_M = C_{Mr} + C_{Ms} \tag{10}$$

C_D, C_L, C_M are considered to represent the sum of static component and rotational component. The total force can be divided into the mean part and fluctuating part, according to the experiment data shown in Tachikawa [6], the mean part is a function of ω/ω_0 and the fluctuating part is a function is a function of both ω/ω_0 and angle of attack α. While in his model, the fluctuating components were tentatively replaced by the static force coefficients $C_{Ds}(\alpha) - \overline{C_{Ds}}$, $C_{Ls}(\alpha)$ and $C_{Ms}(\alpha)$, which was only relevant to the angle of attack α.

Based on the work of Tachikawa [6], but incorporating Kordi's et al. [10] result, a model of lift and drag is proposed in this paper. Those formulas are shown as below:

$$C_{LR} + C_{LS} = A \cdot (1 + 0.32 \cdot (S/S_0) \cdot sin2\alpha)^2 \cdot C_{LS}(\alpha) + B \tag{11}$$

$$C_{DR} + C_{DS} = C \cdot (1 - 0.32 \cdot abs(S/S_0) \cdot cos2\alpha)^2 \cdot C_{DS}(\alpha) + D \tag{12}$$

The first term in the right side represents the fluctuating component, and the second term in the right side represents the mean value. It should be mentioned that the mean value of this fluctuating component is not zero, and the fluctuating part is a function of both S/S_0 and angle of attack α. A, B, C, and D can be solved by the following equations:

$$\mathrm{A} = 2 \cdot \widetilde{f_L}(S/S_0)/(2 \cdot \widehat{C_{Ls}} + 0.12 \cdot (S/S_0)^2 \cdot \widehat{C_{Ls}}) \tag{13}$$

$$\mathrm{B} = \overline{f_L}(S/S_0) - 0.466 \cdot A \cdot S/S_0 \tag{14}$$

$$\mathrm{C} = \widetilde{f_D}(S/S_0)/(\widehat{C_{Ds}} + 0.32 \cdot (S/S_0)^2 \cdot \widehat{C_{Ds}}) \tag{15}$$

$$\mathrm{D} = \overline{f_D}(S/S_0) - \widetilde{f_D}(S/S0) \cdot 0.7 \tag{16}$$

where $S = \omega l/(2U_{rel})$ is the spin parameter of the plate. S_0 used herein is the same as Kordi *et al.* [10]. $\widehat{C_{Ls}}$ and $\widehat{C_{Ds}}$ are maximum value of static lift and drag force coefficients. $\widetilde{f_L}(S/S_0), \widetilde{f_D}(S/S_0)$ are amplitudes of lift and drag coefficients, and $\overline{f_L}(S/S_0)$ and $\overline{f_D}(S/S_0)$ are mean value of lift and drag coefficients as a sum of static component and rotational component, noting that they are all function of S/S_0 and can be deduced from the experiment data from Tachikawa [6]. Their relationships with S/S_0 are shown as below:

$$\overline{f_D}(S/S_0) = \begin{cases} 0.774 + 1.29 \cdot |S/S_0| & |S/S_0| \le 0.35 \\ 1.19 + 0.1 \cdot |S/S_0| & 0.35 < |S/S_0| < 1 \\ 1.29 & |S/S_0| \ge 1 \end{cases} \tag{17}$$

$$\overline{f_L}(S/S_0) = \begin{cases} 2.8 \cdot S/S_0 & -0.1 \le S/S_0 < 0.1 \\ 0.24 + 0.4 \cdot S/S_0 & 0.1 \le S/S_0 < 0.6 \\ 0.8 \cdot S/S_0 & |S/S_0| \ge 0.6 \\ -0.24 + 0.4 \cdot S/S_0 & -0.6 \le S/S_0 < -0.1 \end{cases} \tag{18}$$

$$\widetilde{f_D}(S/S_0) = \begin{cases} 1.1 + 1/3 \cdot |S/S_0| & |S/S_0| \le 0.3 \\ 1.2 + 6/7 \cdot (|S/S_0| - 0.3) & |S/S_0| > 0.3 \end{cases} \tag{19}$$

$$\widetilde{f_L}(S/S_0) = \begin{cases} 0.95 + 5/6 \cdot |S/S_0| & |S/S_0| \le 0.3 \\ 1.2 + 4/7 \cdot (|S/S_0| - 0.3) & |S/S_0| > 0.3 \end{cases} \tag{20}$$

Noting that since there is only data of amplitudes available, so the form of fluctuating is assumed and it is not the unique solution. Then rotational moment coefficient $\mathrm{C_{Mr}}$ is also defined in term of S/S_0, for a rectangular plate, there are three models for C_{Mr} in Tachikawa [6], namely A0, A1, A2, respectively. They are suitable for different initial angles. When S/S_0 is larger than 0.4, those three models are same with each other. In this paper, model A0 and A1 are same as Tachikawa [6], while a modification are made to the model A2. Models A0, A1 and A2 used in this paper are shown as below:

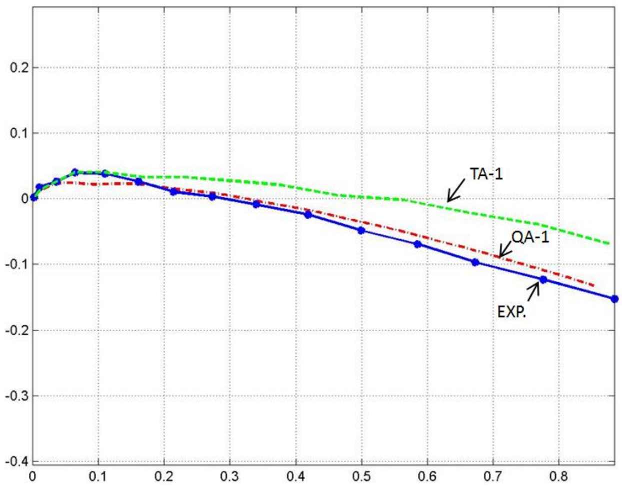
$$C_{MrA0} = \begin{cases} 0.2 \cdot S/S_0 & -0.4 < S/S_0 \le 0.4 \\ 0.08 & 0.4 < S/S_0 \le 0.7 \\ 0.08 - 0.08/0.3 \cdot (S/S_0 - 0.7) & S/S_0 \ge 0.7 \\ -0.08 - 0.08/0.3 \cdot (S/S_0 + 0.7) & S/S_0 \le -0.7 \\ -0.08 & -0.4 \ge S/S_0 > -0.7 \end{cases} \tag{21}$$

$$C_{MrA1} = \begin{cases} -0.02 + 0.25 \cdot S/S_0 & 0 < S/S_0 \le 0.4 \\ 0.02 + 0.25 \cdot S/S_0 & 0 > S/S_0 \ge -0.4 \\ 0.08 & 0.4 < S/S_0 \le 0.7 \\ 0.08 - 0.08/0.3 \cdot (S/S_0 - 0.7) & S/S_0 \ge 0.7 \\ -0.08 - 0.08/0.3 \cdot (S/S_0 + 0.7) & S/S_0 \le -0.7 \\ -0.08 & -0.4 \ge S/S_0 > -0.7 \end{cases} \tag{22}$$

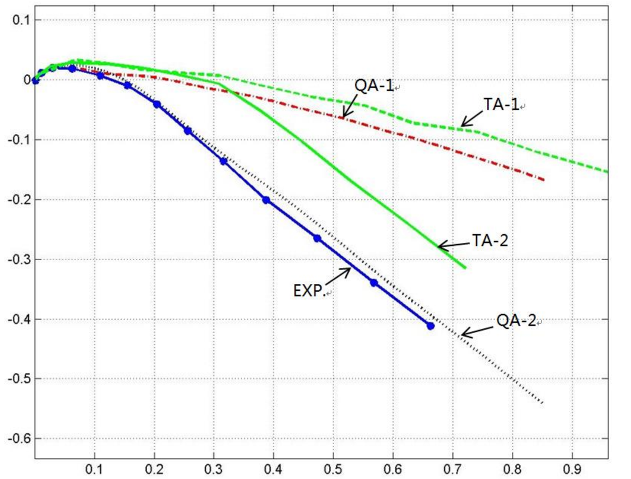
$$C_{MrA2} = \begin{cases} -0.05 + 0.325 \cdot S/S_0 & 0 < S/S_0 \le 0.4 \\ 0.05 + 0.325 \cdot S/S_0 & 0 > S/S_0 \ge -0.4 \\ 0.08 & 0.4 < S/S_0 \le 0.7 \\ 0.08 - 0.08/0.3 \cdot (S/S_0 - 0.7) & S/S_0 \ge 0.7 \\ -0.08 - 0.08/0.3 \cdot (S/S_0 + 0.7) & S/S_0 \le -0.7 \\ -0.08 & -0.4 \ge S/S_0 > -0.7 \end{cases} \tag{23}$$

Results

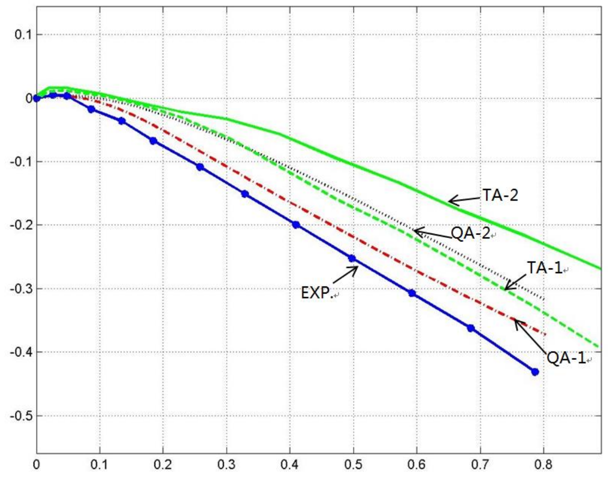
There are currently only two existing test data for the trajectory of a rectangular plate in uniform flow, one is in Tachikawa [6], the other is in Richards *et al.* [9]. While unfortunately, the side view images of Richards *et al.* [9] were vary blurred. So the data of Tachikawa [6] is taken to make a comparison with the numerical results calculated in this paper. The data of Visscher and Kopp [11] is not considered here, because the flow around house has a great effect on the trajectories. Used the stastic and rotational force coefficients defined above, Eqs. 1-3 are solved using a fourth-order Runge-Kutta scheme. Numerical results of trajectories for different initial angles are compared with test data in following figures:



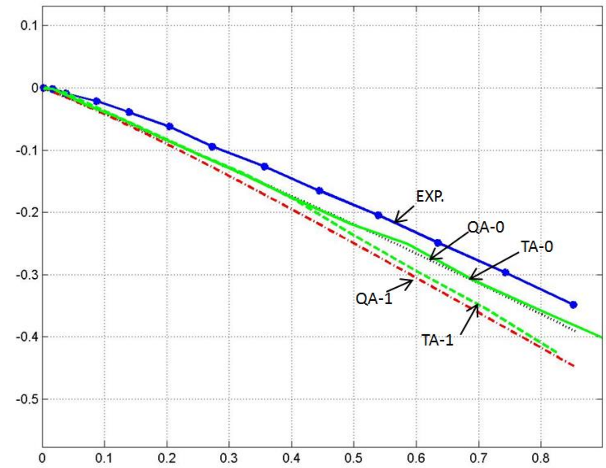
(a)15°(unit:m)



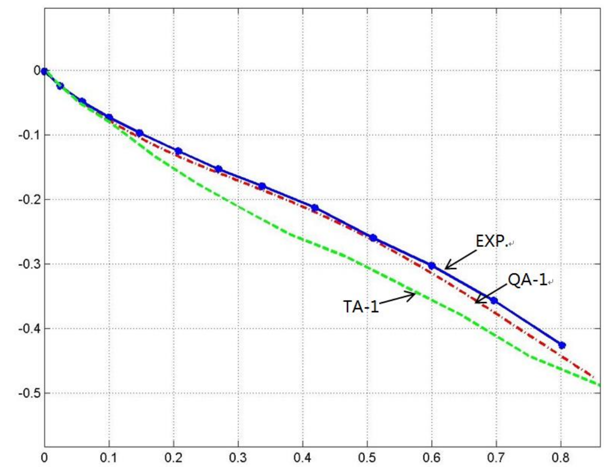
(b)30°(unit:m)



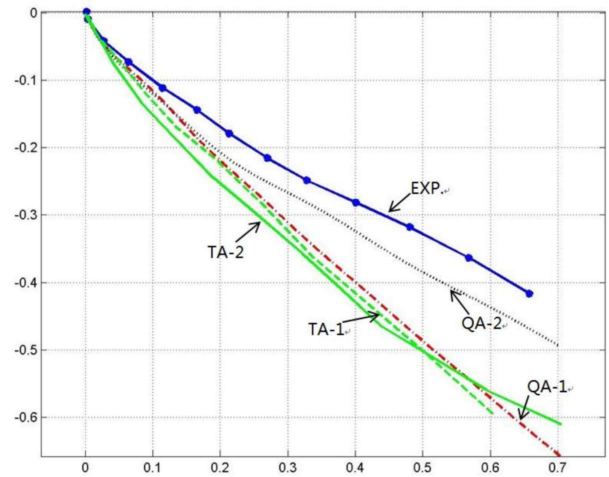
(c)60°(unit:m)



(d)90°(unit:m)



(e)120°(unit:m)



(f)150°(unit:m)

Figure 2. Comparison between the data in Tachikawa [6] and the numerical result calculated in this paper for a rectangular plate with l=40mm, B=80mm, h=2mm, $m/A = 0.224 g/cm^2$, $U_w = 9.2\, m/s$.

Figure 2 shows comparison between the experiment data and the numerical results in this paper for different initial angles, the axis in the Graphs is the position coordinate, the lines in the Graphs are trajectories of the flate plate. TA-0, TA-1, TA-2 in Fig. 2

correspond to calculation results of different rotational moment coefficient models in Tachikawa [6], namely A-0, A-1, A-2 respectively. QA-0, QA-1, QA-2 in Fig. 2 correspond to the calculation results of different rotational moment coefficient models in this paper. EXP. in Fig. 2 means the experiment data in Tachikawa [6]. The results shown in Fig. 11 of Tachikawa [6] can be roughly divided into two groups, one is for plates which have a high rotational speed including case with initial angle of $15°$, the other is for plates which have a low rotational speed including cases with initial angle of $30°, 60°, 90°, 120°, 150°$. For a square plate, the best matches appear to happen in those cases with a low rotational speed, while the worst matches appear to happen in those cases with a low rotational speed in the rectangular plate in Tachikawa [6], especially for initial angle of 30°, 60° and 150°. The numerical calculation results computed by the quasi-steady model proposed in this paper have a better prediction on the trajectory than those in Tachikawa [6] in these cases. While the trajectories in this paper are still scattered compared to the experiment data, this may contribute to two reasons: one is that the Magnus effect should not be taken into account in the early stage of flight, the other may be the imprecise value of c/l. For the case of initial angle of $15°$ which has a high rotational speed in the flight, the numerical calculation result in this paper has a good agreement with the experiment data while the previous model still seems to overestimate the lift and drag force which leads to a higher trajectory. Overall, the quasi-steady model for recctangular plate in uniform flow proposed in this paper provides a better prediction on trajectories than the previous one.

Conclusions

A new quasi-steady model to predict the rotational lift and drag force for rectangular-plate windborne debris in uniform flow is proposed in this paper. Numerical calculations of trajectories of rectangular plate have been made. For the purpose of validation and calibration, the simulated results are compared with the experimental data from Tachikawa [6]. Results show that the numerical trajectories computed in this paper have a better agreement with the test data than those computed in the previous study for all cases. It is suggested that the fluctuating component of rotational lift and drag should be modeled as a function of both angle of attack (α) and dimensionless spin parameter of the plate (S/S_0), ignoring the change of amplitude of rotational lift and drag versus dimensionless spin parameter of the plate (S/S_0) may contribute to the distortion of the numerical trajectories.

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