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Analytical and Experimental Study of a Flat Box Girder Under Bimodal Coupled Flutter Onsets

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Abstract

Flutter characteristics of a flat box girder have been fully investigated by theoretical and experimental analysis. The consistency of aerodynamic derivatives has been studied via the flutter factors that calculated from the tests. The flutter factors have a good agreement with each other and close to the value of a thin plate at different test cases when the section model is under 0 degree attack angle. But the flutter factors decrease significantly and have a great discreteness while the wind attack angle increases. By using different groups of dynamic parameters and fixed aerodynamic derivatives, Flutter factors still showed a good agreement with each other, which is distinct different from the tested results. Which indicated that aerodynamic derivatives is the administered contributor that makes flutter factors' huge discreteness. At last, the analysis on relevance between the phase lag and the flutter factors indicates that phase difference of motion is the main reason which led to the significant change of aerodynamic derivatives of flat box girders. Where there is a bigger phase difference, there is a bigger flutter factor and a higher flutter velocity.

Introduction

Remarkable development of flutter analytical approaches for long span bridges has been made in recent decades. The accuracy of these methods is characterized by the differences between calculated flutter velocity and actually one. However, even under small amplitude oscillation condition, the comparison of flutter onsets between analytical result and wind tunnel test also suggest that errors still exist. And it is well know that if there has a large wind incidence of attack, the errors may be more significant. Some literatures considered that the high order harmonic component may induced these differences. However, Wind tunnel experiments conducted by Wang [5] showed that even if in 10 degree of torsional amplitude, the proportion of high-order harmonic component in aerodynamic forces is still small. Matsumoto [3] conducted a series of wind tunnel tests with two kinds of aspect ratio rectangular models, results suggested that self-excited forces in coupled motions that combined with vertical motion and pitching motion was not equal to the sum of self-excited forces obtained in the independent motions. Lee [4] also founded that aerodynamic forces is significantly depended on the state of coupled motion and phase lag was the key parameter to self-excited forces. Liao [2] verified this phenomenon and also validated the importance of phase lag in coupled motion.

Most of the previous literatures focused on the fact that the coupled motion affects the aerodynamic forces rather than the aerodynamic derivatives. In this paper, a detailed research of the influences that coupled motion of flat box girder affect the aerodynamic derivatives is conducted. A series of wind tunnel tests of a flat box girder section model have been conducted through spring suspension system. With different dynamic parameters, different flutter onset, amplitude, phase lag can be obtained. Based on these results, the flutter factor can obtained through the analytical formula of coupled flutter (Chen, [1]). The variability and the regularity of the flutter factor will be analysed, and then analysed the relationship between phase lag and flutter factor, confirmed the non-consistency of aerodynamic derivatives with different coupled motions and got the conclusion that phase lag would has a great influence on aerodynamic derivatives finally. At last, if have to evaluate the influence of phase lag when conduct aerodynamic derivatives test or flutter analysis, it is a good choice to adopt the method of this paper.

Quantitative Expression of Flutter Performance

Flutter factor is a parameter that reflect self-excited forces in simplified closed form solutions of flutter analysis and has the same function with aerodynamic derivatives. This method based on bimodal coupled flutter mechanism and established a high precision method that can calculate flutter onset using flutter factor and system dynamic parameters. The expression of the simplified closed form solution is given by:

$$U_{cr} = \gamma\omega_{s2}b\sqrt{\left(1-\frac{\omega_{s1}^2}{\omega_{s2}^2}\right)\frac{mr}{\rho b^3}} \tag{1}$$

Where U_{cr} is the critical flutter velocity; ω_{s1} and ω_{s1} is the vertical circular frequency and the torsional circular frequency, respectively; b is the half width of bridge girder; m is the effective mass per unit span; r is the radius of gyration; ρ is the air density; γ is the flutter factor; and the expression of γ is given by:

$$\gamma = 1\Big/\sqrt{F_1+F_2} \tag{2}$$

$$F_1 = (r/b)\,\mathrm{D}^2(-k^2H_3^*)(kA_1^*) \Big/ [(-kA_2^*)+2k\xi_{s2}(1+\upsilon A_3^*)^{\frac{1}{2}}/\upsilon] \tag{3}$$

$$F_2 = (b/r)(k^2A_3^*) \tag{4}$$

where $k=\omega b/U$ is the reduced frequency; ξ is the damping ration in torsional mode; $\upsilon=\rho b^4/I$ and I is the effective polar moment of inertia per unit span; $A_1^*, A_2^*, A_3^*, H_3^*$ are the aerodynamic derivatives. D is the similarity factor in modal shapes of the fundamental vertical and torsional modes, for section model, D=1.

Then the final critical velocity is the intersection of the curve by equation (1) as a function of reduced frequency k with the curve by equation (5) that is given by:

$$U_{cr} = (1 + \upsilon A_3^*)^{\frac{1}{2}} \omega_{s2} b / k \qquad (5)$$

For a certain dynamic system, it is easy to know that flutter factor γ is just a function of aerodynamic derivatives from equation (1) to equation (5). According to the conclusion that aerodynamic derivatives are functions of reduced velocity and independent of states of motion in small amplitude oscillations, it is easy to conclude that for a same section and a same wind incidence of attack, even different type of dynamic systems and different forms of motions, the flutter factor will not have a significant changes because the aerodynamic derivative is the same.

However, this study discovered significant differences of flutter factors in different states of flutter motions through section model wind tunnel tests, and also founded that the phase lag between vertical motion and pitching motion would has a great influence on flutter factor or aerodynamic derivatives at large incident angles.

Section Model Test for γ

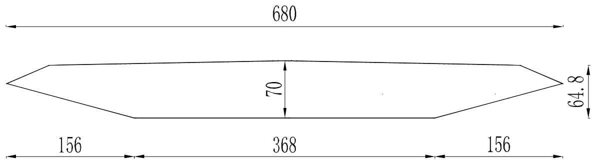

Figure. 1 Cross-Section of model (unit: mm)

The flat box girder section model has a height of 0.07m and a width of 0.68m and a length of 2.095m. The cross section of the model is shown in figure 1 and the two degrees of freedom spring suspension system is shown in figure 2.

Figure. 2 Test model in wind tunnel

Scruton number Sc of the system was varied in wind tunnel tests in order to obtained different flutter critical velocities, amplitudes and forms of flutter motions, and Sc is defined as follows.

Sc for vertical motion:

$$Sc_\eta = \frac{2m\delta_\eta}{\rho H^2} \qquad (6)$$

Sc for torsional motion:

$$Sc_\alpha = \frac{2I\delta_\alpha}{\rho H^4} \qquad (7)$$

Where δ_η, δ_α is the logarithmic decrement for vertical system and torsional system, respectively; H is the height of the girder. The test changed the parameters m and I to get different Sc numbers to get different critical velocities. All tested cases are listed in table 1.

Case	m	I_m	f_h	f_t	ξ_{s1}	ξ_{s2}	η
Case1	22.12	0.685	1.63	3.00	0.470	0.253	0.54
Case2	20.14	0.681	1.75	3.06	0.485	0.287	0.57
Case3	22.12	0.780	1.63	2.81	0.476	0.298	0.58
Case4	20.14	0.755	1.75	2.94	0.456	0.312	0.60
Case5	20.14	0.861	1.75	2.75	0.462	0.265	0.63
Case6	22.12	0.987	1.63	2.50	0.476	0.304	0.65

Table. 1 Testing cases for flutter of sectional model

Results of Wind Tunnel Tests

Based on the dynamic parameters of Case1-Case6 and the tested critical velocities, it is easy to obtain the flutter factors corresponded to each cases according to equation (1). The result of flutter factors are shown in table 2.

To analysis the discreteness of flutter factor and investigate the reason that led to the discreteness, here proposed a parameter that called RC(relative change) and the expression of RC is given by:

$$RC = \frac{|\gamma - \gamma_{\max}|}{\gamma_{\max}} \qquad (8)$$

According to equation (8), the relative change RC of flutter factor can easily been obtain, as list in table 2. From table 2, for Case1-Case6 under 0 degree incidence of attack, the flutter factor has a very small RC, but with the increase of α, the RC is bigger. For 0 degree incidence of attack, γ ranging from 0.407 to 0.426, and the RC is only 4.5%. It is appropriate to conclude that the aerodynamic derivatives didn't have a significant change and the dynamic parameters can't led to a significant change of γ in this wind incidence of attack. But for 3 or 5 degree incidence of attack, the discreteness of γ is bigger than the former, and the biggest RC can reach to 11.8%. It is evident that when the girder is become more like a bluff body, the RC of flutter factor is more significant. Then it is necessary to study that what have caused the big RC of the flutter factors.

Angle of attack α (deg)	Case	r/b	f_t	$\frac{f_t}{f_h}$	Reduced velocity	Flutter factor γ	RC
0	Case 1	0.518	3.00	1.85	8.62	0.418	1.9%
	Case 2	0.541	3.06	1.77	7.99	0.412	3.3%
	Case 3	0.552	2.81	1.73	7.96	0.407	4.5%
	Case 4	0.569	2.94	1.68	7.99	0.421	1.2%
	Case 5	0.608	2.75	1.57	7.65	0.426	0
	Case 6	0.621	2.50	1.54	7.69	0.420	1.4%
3	Case 1	0.518	3.00	1.85	8.08	0.391	4.4%
	Case 2	0.541	3.06	1.77	7.20	0.373	8.8%
	Case 3	0.552	2.81	1.73	7.83	0.391	4.4%
	Case 4	0.569	2.94	1.68	7.45	0.392	4.2%
	Case 5	0.608	2.75	1.57	7.36	0.409	0
	Case 6	0.621	2.50	1.54	7.44	0.402	1.7%
5	Case 1	0.518	3.00	1.85	7.24	0.353	7.3%
	Case 2	0.541	3.06	1.77	6.31	0.336	11.8%
	Case 3	0.552	2.81	1.73	6.65	0.340	10.8%
	Case 4	0.569	2.94	1.68	6.74	0.367	3.4%
	Case 5	0.608	2.75	1.57	6.89	0.381	0
	Case 6	0.621	2.50	1.54	6.75	0.373	2.1%

Table. 2 Calculated results of flutter factors in Case1-Case6

Influence of the Parameters on γ

Effects of Dynamic Parameters on γ

The major parameters that have influence on γ is the dynamic parameters (the non-dimensional parameter *r/b*; the non-dimensional effective polar moment of inertia per unit span *I*; the reduced frequency *k*; the damping ratio ξ *et, al*) of the system and the aerodynamic parameters (A_i^*, H_i^*) of the section. To study the sensitiveness of flutter factor to dynamic parameters, aerodynamic derivatives under different wind incidence of attacks have to been fixed and use different groups of dynamic parameters, then calculate the flutter factors by using equation (2). If the dynamic parameters have a significant influence on the flutter factor, γ will have a remarkable change, otherwise, the *RC* of γ is mainly due to the change of aerodynamic derivatives. Forced vibration experiment has been conducted in XNJD-1 wind tunnel to identify the aerodynamic derivatives, by forced to heaving and pitching pure motion under different wind incidence angles of attack. The identified aerodynamic derivatives under different angle of attacks of this girder are shown in figure 3. According to the results of experimental aerodynamic derivatives, figure 4 gives the regularity of flutter factor changing with reduced velocity when using six groups of dynamic parameters (Case1-Case6).

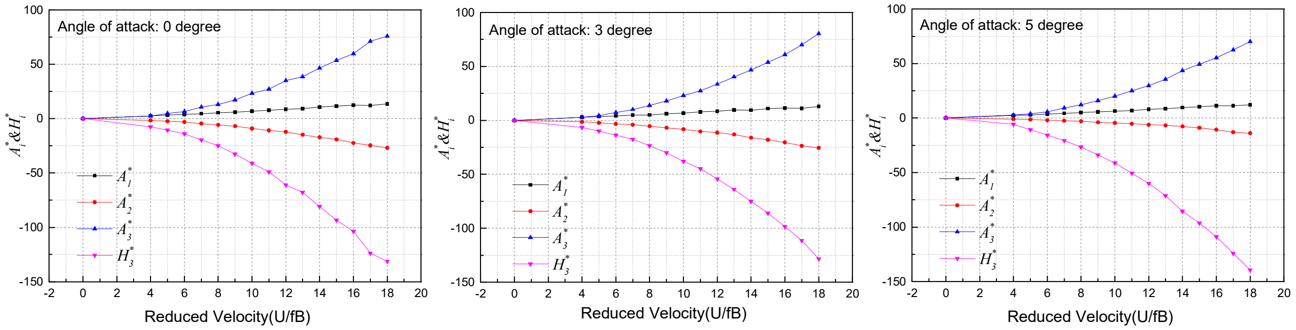

Figure.3 Aerodynamic derivatives of the girder under different wind attack.

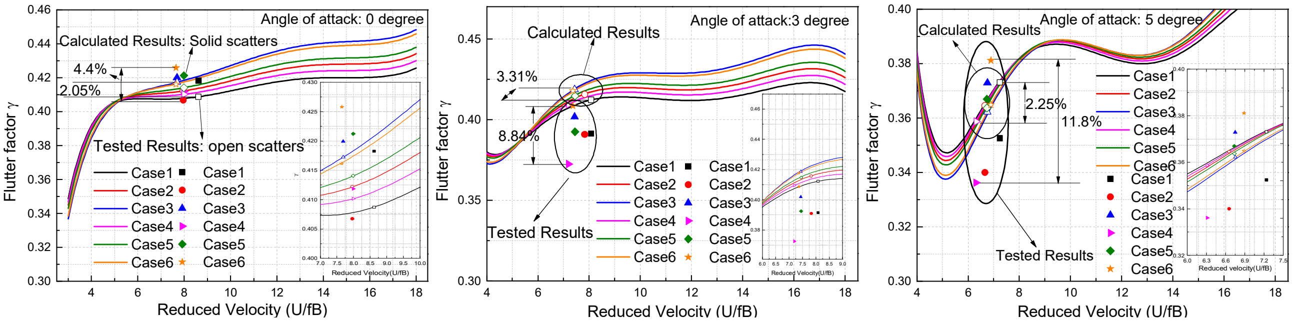

Figure.4 The regularity of flutter factor γ changed with reduced velocity

Where the solid point is the flutter factors that obtained from experiments, whereas the hollow points and the lines are results of flutter factors that obtained from calculation. It is obviously that even with different sets of dynamic parameters (Case1-Case6), the *RC* of γ is mostly less than 5%. That can be concluded that if the aerodynamic derivatives is fixed, γ will not has a significant change even if the dynamic parameters changed. But in wind tunnel test, the *RC* of γ can even reached to 11.8%, it is clear that the change of the aerodynamic derivatives induced the significant *RC* of flutter factor γ.

Effects of Phase lags on γ

Then, the phase lag in coupled motion of each case has been extracted and then the relationship between flutter factor and phase lag has been studied. To provide an overall view of the relationship between flutter factor and phase difference, the curve is given in figure 5. For Case1-Case6 under 0 degree angle of attack, the flutter factor has a little fluctuation and almost independent on phase lag, and the aerodynamic derivatives also independent on forms of motion. But under 3 or 5 wind incidence angle of attack, the flutter factor increases while the phase difference increases, and the flutter factor show an obviously discreteness. That means that the phase difference could have great influence on flutter factors and aerodynamic derivatives when the girder is more like a bluff body rather than a streamline body.

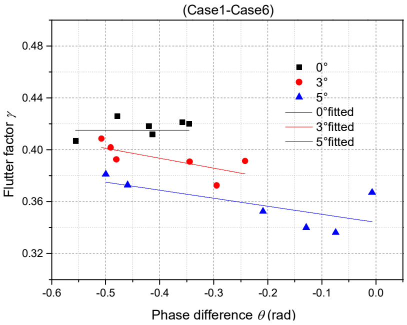

Figure.5 Comparison of flutter factors and phase differences

Conclusions

Through the experimental and theoretical analysis of flutter performance of a flat box girder under different wind incidence angle of attack, it is easy to obtain some conclusions as follows:

(1) At the 0 degree angle of attack, the flutter factor centre on a certain region and the *RC* less than 5%, but the *RC* of flutter factor will increase significantly when the angle of attack increases.

(2) There has a positive relationship between phase lags of coupled motion and flutter factors. The flutter factor increases while the phase lag increases and phase lags will have a great influence on flutter factors when bridge girder has a big wind incidence angle of attack.

(3) The main reason that causes the discreteness of flutter factors is not the dynamic parameters but the remarkable change of aerodynamic derivatives. That means the change of phase difference of motion may cause the variation of flutter derivatives. So, the influence of phase difference should be taken in consideration in flutter analysis or aerodynamic derivatives testing.

Acknowledgments

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