Measuring if things group the way I intended: Factor analysis

Prof Gavin T L Brown, *The University of Auckland*, Seminar for the BVM group, Applied Educational Science, Umea Universiteit, Sept, 2016

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Problem of depending on single variables

- Lack of reliability in measurement with single variable
  - Objective demographic variables can probably be measured accurately with single prompt—
    - e.g., SEX, AGE, ETHNICITY?
  - Opinion/Attitude Proficiency/Ability variables highly susceptible with single measure
    - How overcome?: Multiple measures of same thing
    - How do we know they are the same?: Factor Analysis

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Logic of Factor Analysis

- If an opinion is real and stable
  - the mathematical pattern of responses to items related to the opinion should be highly correlated
  - there should be little Noise or Error or Residual unexplained after the pattern is found
  - Correlations between items of same factor should be higher than their correlations with items they are not related to

  - Note the similarity to reliability theory in CTT

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Effect of a Factor

- The factor simplifies the data
  - Data reduction
  - Instead of reporting multiple items, we report one factor score and improves confidence in interpretations
- The weight of multiple measures ensures better estimation of opinion or attitude or ability
  - The factor reduces chance effects
  - The factor reduces error

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Getting to a Factor

- Direct observation/measurement
  - Answers to test questions
  - Response to rating statements
  - Measures of volume, mass, area
- Indirect inference about latent trait or ability
  - Factors that are derived from the statistical interaction of directly observable measures
  - *Is it real, if it cannot be directly observed?*
    - Is evolution not real just because it happens on a time scale too difficult for us to observe?

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Factors

- Latent
  - derived from variables that group in common;
  - not directly observed
- Efficient—one value instead of multiple values
- Increased Accuracy—multiple measures reduce error

Factors—Spearman (approx 1900)

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- Means of simplifying data by examining the commonalities in a matrix of data
- Values for response can be different for cases
- Pattern of responses across number of variables detected show pools of variables

	Variables			
Case	*A*	*B*	*C*	*D*
a	8	7	5	8
b	4	3	2	4
c	7	6	4	7
d	3	2	1	3

We expect A, B, & D to be in same factor

Factors

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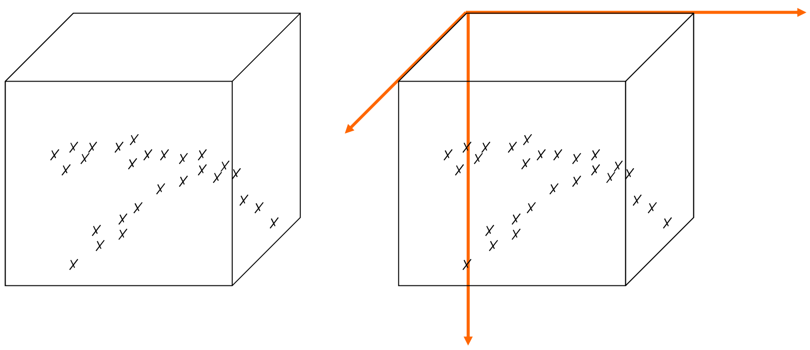
- Vectors through *n*–dimensional space
  - Angles that meet at the ceiling of a square room are vectors perfectly orthogonal (90º to each other)



NB: Correlation of factors $r = 0.00$ when orthogonal

Factors

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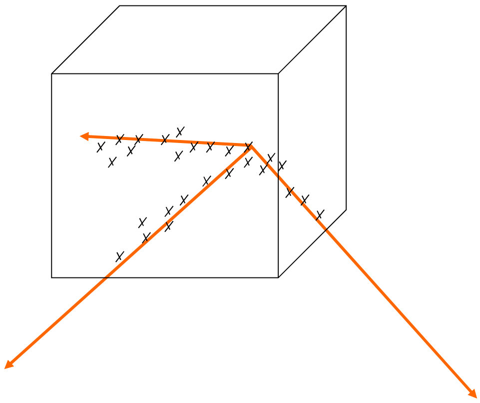
- Rotation—we find a point from which we can see the vectors along which the variables group, minimise difference to lines (similar to regression)



Factors

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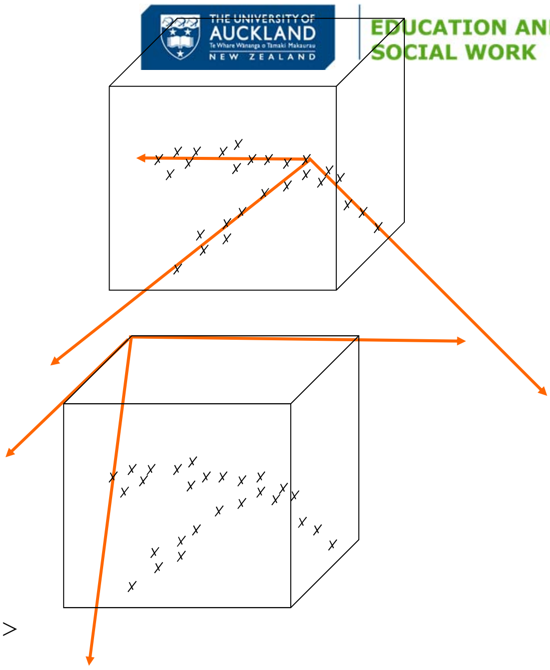
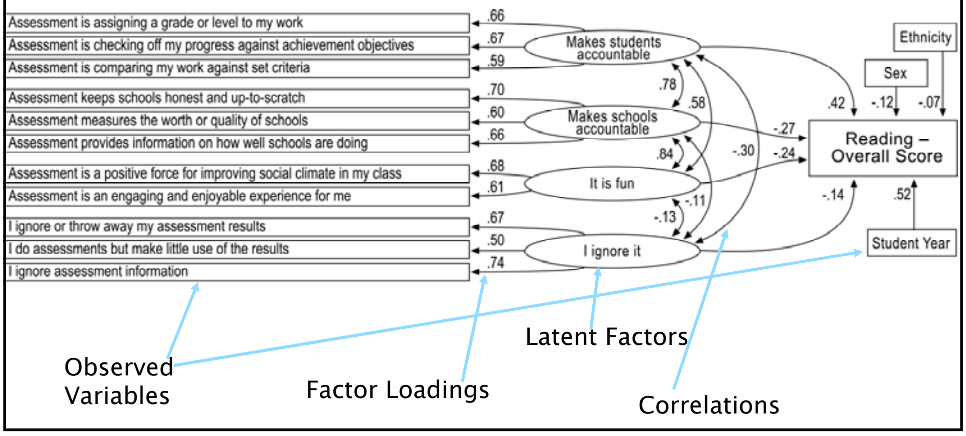
- Correlated—psychological artefacts are likely to be oblique or connected with each other unlike an orthogonal room



NB: Correlation of factors *r* > 0.00 when oblique

Factor Example: Variables, Factors, Correlations



Best Factor Analysis

- Maximum Likelihood Estimation (MLE)
  - Best estimate of population values
  - Not principal components which includes error
- Oblique Rotation
  - Not orthogonal
- At least 3 items per factor
  - Loading >.30
- At least 5 cases per item
  - Preferably 15
  - But if large numbers of items in a factor, number of cases can be low: 12 items←→50 cases

Bandalos, D. L., & Finney, S. J. (2010). Factor analysis: Exploratory and confirmatory. In G. R. Hancock & R. O. Mueller (Eds.), *The reviewer's guide to quantitative methods in the social sciences (pp. 93-114). New York: Routledge.*
Costello, A. B., & Osborne, J. W. (2005). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Practical Assessment Research & Evaluation, 10(7).*

Math attitudes

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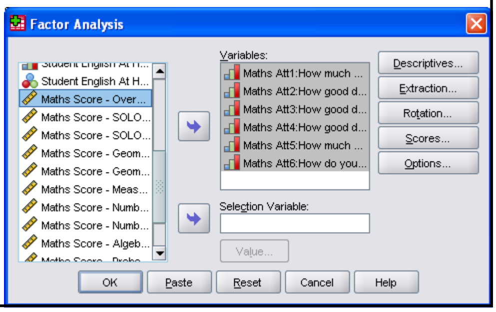
- 6 items too many. Can they be reduced to a smaller number of meaningful categories?
- Analyse>Dimension Reduction>Factor Analysis
  - Options:
    - Extraction MLE; eigenvalues>1
    - Rotation Oblimin



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Total Variance Explained

Factor	Initial Eigenvalues Total	Initial Eigenvalues % of Variance	Initial Eigenvalues Cumulative %	Extraction Sums of Squared Loadings Total	Extraction Sums of Squared Loadings % of Variance	Extraction Sums of Squared Loadings Cumulative %	Rotation Sums of Squared Loadings[a] Total
1	2.948	49.134	49.134	2.469	41.143	41.143	2.060
2	1.036	17.263	66.397	.549	9.147	50.290	2.134
3	.643	10.710	77.107				
4	.516	8.603	85.710				
5	.460	7.665	93.375				
6	.397	6.625	100.000				

Extraction Method: Maximum Likelihood.
a. When factors are correlated, sums of squared loadings cannot be added to obtain a total variance.

Goodness-of-fit Test

Chi-Square	df	Sig.
51.159	4	.000

Factor Correlation Matrix

Factor	1	2
1	1.000	.611
2	.611	1.000

Extraction Method: Maximum Likelihood.
Rotation Method: Oblimin with Kaiser Normalization.

Pattern Matrix[a]

	Factor 1	Factor 2
Maths Att1:How much do you like doing maths at school?	.824	.026
Maths Att5:How much do you like doing maths in your own time (not at school)?	.692	-.068
Maths Att6:How do you feel about doing things in maths you haven't tried before?	.489	.111
Maths Att3:How good does your teacher think you are at maths?	-.019	.731
Maths Att4:How good does your Mum or dad think you are at maths?	-.028	.700
Maths Att2:How good do you think you are at maths?	.103	.696

Extraction Method: Maximum Likelihood.
Rotation Method: Oblimin with Kaiser Normalization.
a. Rotation converged in 7 iterations.

Factors are only real when they have strong theoretical and statistical evidence.

Are these 2 factors interpretable? What shall we call them?

How many factors?

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- Optimal Coordinate
  - Measures gradients associated with eigenvalues and their preceding coordinates
- Acceleration Factor
  - Determines coordinate where the slope of the curve changes most abruptly
- Velicer's Minimum Average Partial (MAP)
  - average squared correlations after partialling out previous components
  - use polychoric correlations for ordinal variables
- Horn's Parallel Analysis
  - takes into account the proportion of variance resulting from sampling error
- Ruscio & Roche's Comparison Data
  - multiple datasets with known factorial structures are analyzed to determine which best reproduces the profile of eigenvalues for the actual data

Strong

- Kaiser's eigenvalue-greater-than-one rule (K1)
  - PCA not FA
  - Tends to overestimate number of factors
- Cattell's (1966) scree test
- eye-ball sree plot of eigenvalues for a break, hinge, or "elbow"
  - Highly subjective, low inter-rater reliablity
- Very Simple Structure
  - Only if very few factors in data

Weak

Number of Factors

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- SPSS + R plug-in to calculate
- Courtney, M. G. R. (2013). Determining the number of factors to retain in EFA: Using the SPSS R-Menu v2.0 to make more judicious estimations. *Practical Assessment Research & Evaluation, 18(8), Available online: http://pareonline.net/getvn.asp?v=18&n=18.*

Table 1. Summary of Modern Techniques for Determining Number of Factors to Retain in EFA

Modern Technique	Standard for all Data Types	% Accuracy	Bias in simulation	Recommended version for ordinal data
CD	CD_r	87.14	Slight under-extraction	CD_{r_s}
PA	PA-PCA_{rm}	76.42	Unbiased	PA-$PCA_{\rho m}$
OC	OC_r	74.03	Slight under-extraction	Not established
MAP	$MAPr^2$	59.6	Moderate under-extraction	$MAP\rho^2$
AF	AF_r	45.91	Substantial under-extraction	Not established

Note. Accuracy and Bias estimates taken from Ruscio & Roche's (2012) simulation study (p. 289). Although the OC and AF procedures may be carried out with Spearman or polychoric correlations in the SPSS R-menu v2.0, such modifications are not established.

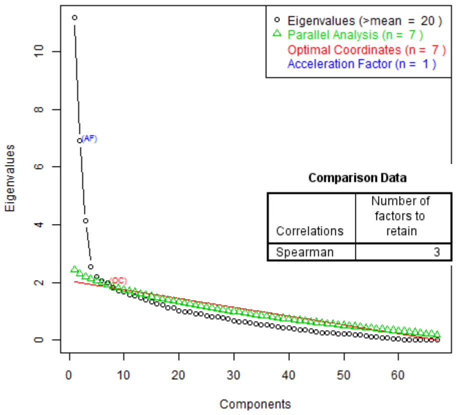
Number of factors

- 67 items from 2 inventories, total 8 starting factors
- Possibly 1–7 factors
  - All options between 3–7 tested
  - 4 was most interpretable in terms of item content per factor

Velicer's Minimum Average Partial Test

	Velicer's Minimum	
	Minimum	Components to retain
Squared MAP	.013	4
4th power MAP	.001	4

Parallel analysis on data permutation



Once you have a factor….

- How can you be sure it is right?
- What evidence can we use?
  - Maximum likelihood estimation permits tests as to whether the observed result fits the data
- Confirmatory factor analysis tests EFA result by setting all off-paths to ZERO

Mathematical Test of Model

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- CFA is a sophisticated correlational-causal mathematical testing of a model against a data set
- Does the model even solve properly?
- How close are they? Does the model fit the data?
  - Models are rejected if they do NOT have close fit to the data
    - the data can't be wrong esp. if it is a representative and large sample—it's the reality we are trying to model
  - Models are NOT accepted if they have close fit to the data
    - They are NOT YET DISCONFIRMED—Popper
    - Multiple models can fit equally well the same data
    - Fit could be attributable to chance factors in the data we collected

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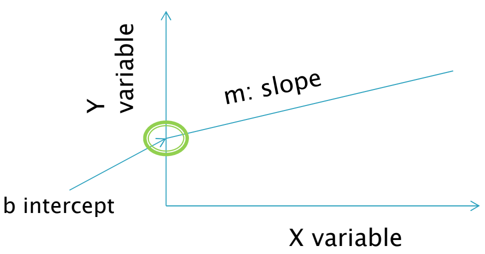
The linear relationship

- Changes in XXX cause a linear change (increase or decrease) in YYY
- Formula:
  $Y = m*X + b$
  - m=slope [standardised beta = a proportion of standard deviation]
  - b=intercept [starting point of equation; represents tendency to respond]
- Multiple predictors
  - $y = b_0 + b_1X_1 + b_2X_2$….. (just keep adding an X for each new variable)



Interpretations:
1. For every 1 *SD* change in X, you will get m**SD* change in Y.
2. This relationship explains *x*% of variance in Y

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Looking Under the Hood: Components of CFA and SEM models

- Variables
  - Manifest [observed behaviours, usually dependent, rectangles]
  - Latent [unobserved, explanatory, ovals]
  - Residual [unobserved, unexplained, ovals]
- Manifest variables are predicted by both Latent traits and residuals
  - Goal to have large proportion of variance in manifest explained by latent rather than residual disturbances

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Looking Under the Hood: Components of CFA and SEM models

- Paths
  - **Fixed**: equations require SEED values to solve; 1 is the conventional seed. All latent traits must have one path to their predicted manifest variables with a fixed value. All other values are estimated relative to the seed value.
  - **Free**: All other paths are allowed to be estimated freely based on the data provided to the model; they may be stronger than the fixed path, but better to make the strongest path in a factor the fixed path.
  - **Zero**: Paths not required by the model are forced to be non-existent. This contrasts to EFA where all paths have some freely estimated value.

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Estimation

- Maximum likelihood (most common)
  - The parameter values in the data set (a sample) are the most likely values in the population (not present, but to which we wish to generalise)
  - Hence, procedure attempts to maximise the input values when estimating the solution
    - means, standard deviations, covariances
  - Hence, it matters that the sample reflects the population and is sufficiently large that parameters are likely to apply to population
  - Valid if response categories are defensibly continuous (i.e., ≥ 5 ordinal categories)

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Model Evaluation: Fit to Data

- Because of MLE, it is possible to evaluate the fit of the model relative to the data by comparing the distributions
  - The chi-squared (χ^2) test is the fundament of model evaluation
  - χ^2 test: difference between Observed (model) and Expected (Data) adjusted by number of parameters and cases (degrees of freedom)
  - However, $\chi 2$ penalises **falsely** large N (i.e., >100) and large number of manifest variables
  - So it is a poor test, notwithstanding vehement objections by some researchers

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Evaluating Results: Which Fit indices & What Values?

	Goodness of Fit		Badness of fit	
Decision	*p* of χ2 / *df*	CFI gamma hat	RMSEA	SRMR*
Good	>.05	>.95	<.05	≈.06
Acceptable	>.05	>.90	<.08	<.08
Marginal	>.01	.85-.89	<.10	
Reject	<.01	<.85	>.10	>.08

Note.
Report multiple indices but beware…..
CFI punishes **falsely** complex models (i.e., >3 factors)
RMSEA rewards **falsely** complex models with mis-specification

See Fan & Sivo, 2007

*AMOS only generates SRMR if NO missing data;
thus, important to clean up missing values prior to any analysis.
Recommend expectation maximization (EM) procedure

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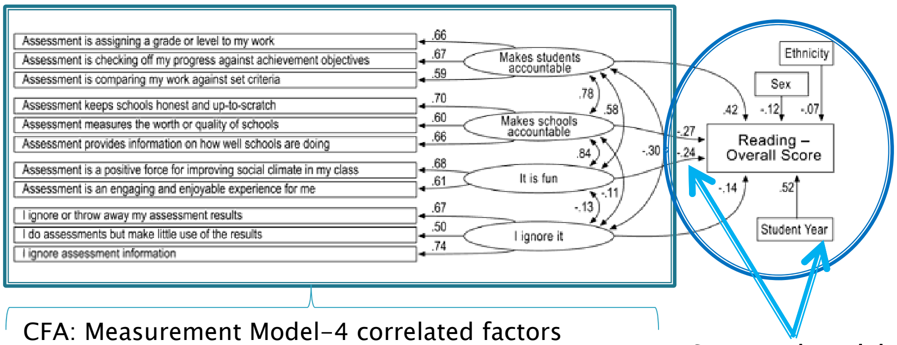
CFA + SEM

(Brown & Hirschfeld, 2008)



CFA: Measurement Model-4 correlated factors
Note. Accurate measurement models are also needed for reading score, year, sex, & ethnicity

Structural model: multiple predictors of performance

Note.
If measurements of each construct are **NOT** robust, do **NOT** use them for anything!!!

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What is Confirmation in CFA?

- Most studies follow this process
  - An inventory is developed using theory
  - The validity of the questionnaire may be explored
  - EFA identifies a plausible model within a data set
  - CFA tests the fit of the EFA model to the data
  - CFA refines the EFA model with the same data
  - This process is better considered Restrictive analysis not CFA
- True confirmation comes when an existing model is TESTED with an independent sample
  - Requires that 2nd sample is drawn from the same population
  - No EFA needed
  - Just run the model, does it fit?
  - If NOT, then EFA must begin again…

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Application to testing

- Do all the items belong to one dimension? Is the test unidimensional?
  - An advanced form of discrimination checking
  - If yes, IRT analysis appropriate
  - If not, then determine dimensionality and test robustness and validity with CFA and IRT
  - report dimension scores
- If you planned sub-tests then you want multiple dimensions. But did you get the dimensions you wanted? As many as you planned? In the types you planned?
  - Test with CFA for the desired structure
  - If robust, then report sub-test scores

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Available from