NAVIGATING FLIPPED LEARNING: INSIGHTS FROM A GRADUATE-LEVEL ALGEBRAIC GEOMETRY COURSE

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This study explores the integration of flipped learning into a graduate-level algebraic geometry course, addressing gaps in understanding its implementation at this educational level. Through an exploratory case study, students' experiences were examined, and thematic analysis revealed that students had nuanced perceptions of this integration with four major themes arising: Preparation and Workload, Content Interaction, Social Interaction, and Resources. While students appreciated collaborative aspects and the emphasis on problem-solving, challenges emerged, including an increased workload and a strong preference for explicit forms of instruction. This research underscores the need for further exploration to refine flipped learning practices and gain a comprehensive understanding of its implications on student experiences in graduate mathematics education.

INTRODUCTION

Since its emergence, flipped learning has garnered much attention from researchers and practitioners, acknowledged for its ability to enhance inter-student interactions through dynamic and collaborative practices (Bergmann & Sams, 2012). As a mode of instruction, it challenges the traditional classroom norms and supports a more dynamic environment where students take on more responsibility in their learning. In this setting, students are expected to engage with mathematical content before meeting with the instructor and their peers. However, in practice, successful implementations of an instructional mode can be complicated and met with many challenges (Lo et al., 2017). It relies on the responsibilities of teachers and students alike, and more work is needed to understand its implementation at the graduate level.

This study reports on an attempt to integrate aspects of flipped learning into a graduate-level mathematics course. By infusing elements of flipped learning into a traditionally lecture-based course, it was assumed that the limitations of both modes of instruction could be addressed and mitigated. This exploratory case study sought to explore how flipped learning can be realised in a largely untouched context at the highest level of tertiary maths education by considering student perspectives. To this end, we pose the following research question: How do students perceive aspects of flipped learning in a graduate mathematics course?

FLIPPED LEARNING

Flipped learning is positioned in a broader model of learning: blended learning. Despite numerous definitions posed across the literature over time (Bishop & Verleger, 2013), we adopt a common definition used by the Flipped Learning Network (FLN) (2014),

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which describes it to be ‘a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space.’ Their framework outlines the four main pillars that must be incorporated into practice before it is considered flipped learning: Flexible Environment – the importance of being able to provide varied ways of engaging with the content; Learning Culture – the active role students have and evaluate their learning; Intentional Content – careful consideration of the material to be covered independently and in class; and Professional Educator – the essential role of the instructor in the classroom to provide feedback, monitor student progress and the practices in the lesson.

The reported effects of flipped learning on learning achievement in mathematics generally demonstrate a positive trend and highlight various advantageous outcomes, including improved opportunities for feedback, interactions, and applying concepts in class (Lo et al., 2017). Similar findings were echoed by Cevikbas and Kaiser (2022), who found that the reported opportunities offered by a flipped classroom approach were often related to conceptual gains, engagement, and collaboration. On the other hand, the challenges of a flipped learning approach reflect what happens within the lessons themselves and outside of them. Cevikbas and Kaiser (2022) report on four main groups of challenges: pedagogical (e.g., lack of preparation by students), technical (e.g., poor internet connectivity), cognitive (e.g., difficulty remembering lecture video content), and affective issues (e.g., lack of motivation). Students’ unfamiliarity with the approach and the high investment required by instructors are two further oft-reported challenges (Lo et al., 2017). Flipped learning in mathematics has been researched extensively; however, much insight stems from research at the undergraduate level (Lo et al., 2017). Our exploratory study aims to fill a critical gap in the graduate mathematics literature by exploring the integration of flipped learning into a graduate-level course and providing insights into student experiences and perceptions.

METHOD

Research Design

An exploratory case study research design was employed to explore the implementation of flipped learning into our course of interest. This was suitable as this course offered an opportunity to investigate a revelatory case, a phenomenon in a context that had not been explored before (Yin, 2018, p. 50).

Context

The course within this study was a graduate-level algebraic geometry course at a large research university in New Zealand, with a particular focus on curves. Most iterations of this course see 10-15 students enrolled. The instructor assigned a set of YouTube videos created by a Fields medallist, which followed the structure of a traditional lecture and were around 20 minutes each. The course instructor kept some of the class time for revisiting the concepts and ideas from the videos and usually would spend about 20 minutes (out of 50) providing a more traditional lecture on the whiteboard addressing any complex or critical ideas. The course had various aspects of flipped learning implemented into it: the offering of multiple ways to engage with the content through many resources and activities (Flexible Environment); the greater time students had to engage with problem sheets and discuss the content (Learning Culture); the intentional assignment of resources and the curation of complementary problem sheets (Intentional Content); and the availability of the instructor during the lessons, both to provide feedback and monitor ongoing student progress (Professional Educator).

Participants and Data Collection

About half of the students were graduate students, while the other half were undergraduate students who had obtained special permission to enrol in the course. Of the fourteen students in the course, twelve participated in this study. All the students involved were studying mathematics as part of their academic programmes, while many students were also immersed in related fields such as physics, statistics, and finance. All students were sent surveys to complete via Qualtrics on the penultimate teaching week of the second semester of 2023. The survey contained questions about student background, affective factors, course engagement, and various open-ended questions exploring student perspectives of the course. The surveys were conducted to gauge the student perspectives on their experiences on several aspects of the course.

The analysis of this study involves student responses to four open-ended questions: (1) ‘What are the aspects you liked about this course or what aspects did you dislike?’, (2) ‘Regarding the flipped learning aspect of the course, how did you think this affected your experience for the course?’, (3) ‘What would you like to see improved in a course like this?’, (4) ‘What is your preferred learning model you prefer and why? (e.g., flipped lectures, traditional lectures, online lectures, a mix, etc.)’. The open responses allowed participants to elaborate on more nuanced aspects of their experiences than can be seen by closed-response questions alone.

Analysis

Thematic analysis was used to identify relevant themes from the open response data. The thematic analysis approach used in this study was inductive, where themes and codes were driven from the data rather than selected a priori. A thematic analysis provides a helpful way to view qualitative data, uncover new perspectives, and identify the similarities and differences between participant responses.

We engaged in a complete coding process where the open-response survey data from all twelve participants were read, and all potential features of the data were coded before engaging in further iterative processes. While we have tried to avoid actively construing meaning in a way that is unsupported by the responses, there is a degree of interpretation required in selecting codes and themes. Our active position as researchers within this process cannot be avoided, and we recognise the inevitable ever-present bias in any form of research. Any codes irrelevant to addressing our research question were omitted from the analysis.

FINDINGS

The comments directly related to the flipped aspects of the course were nuanced, with different dimensions and preferences being voiced by various students. Participants frequently provided balanced views in these comments; however, some clearly articulated dichotomous comments—either positive or negative—regarding the experience. Many comments were quite assertive, with a few stating that flipped learning does not work for a demanding course. As one student said, ‘It was a difficult course unfortunately and thus required better instruction than could be provided from flipped lecture’ (student 4). Others noted that the experience was nice, unexpected, and comparable to traditional modes of instruction in difficulty. From the thematic analysis, four major themes regarding how students perceive flipped learning aspects in the course were identified from the data: *Preparation and Workload*, *Content Interaction*, *Social Interaction*, and *Resources*. In the following section, each theme is explained, and extracts illustrating themes are provided.

Preparation and Workload

The theme of *Preparation and Workload* captured codes related to one of the major differences between a flipped classroom and a more traditional instructional approach, which is the work required by students to prepare for classes. Students are required to engage with content before a lesson, unlike a traditional lecture where content engagement typically happens for the first time during the lecture.

Students attributed the higher-than-usual workload to the preparation needed to be done for the lessons, and sometimes this increase was perceived to be significant: ‘The current set-up with flipped lectures did dramatically increase the workload of the course by having recordings to watch outside of the lecture times’ (student 8). Additionally, this preparation was seen by some students as being essential to participating or even simply attending the lectures. For some, a lack of proper preparation meant it would be ‘very easy to fall behind’ (student 4). One participant noted that they ‘Missed out much of the opportunity to practice solving problems when fallen behind on lectures’ (student 7). For others, it was the case that preparation, or the lack thereof, impacted their attendance. One student expressed their views as such: ‘I was more motivated to keep up to date with the lecture content. When I haven't kept up to the content, I was less likely to show up to the lectures’ (student 7). This student reported that the flipped learning aspect had encouraged them to stay up to date with the current workload but that when they slipped behind, they felt less inclined to attend the lectures. Falling behind did not seem uncommon among the participants, which is not ideal considering that the flipped aspects were opportunities for students to apply the knowledge and skills they have encountered in different contexts. Furthermore, with many concepts being so highly connected, it is unlikely that this will have no bearing on a student’s ability to engage with later content. Unfortunately, this appears to have been the case for one student who reported this to be a problem for them across the semester: ‘I was not able to consistently watch the recordings due to the increased work load of the course, and as a result I ended up falling behind in the content for most of the course' (student 8).

Content Interaction

Content Interaction concerns comments coded for relevance to how students engage with the learning material. This may be through the tasks during class, such as, most often mentioned, solving exercises in class, but comments under this theme also reflect the difficulties students had in keeping up with more demanding content and a desire to revise fundamental knowledge.

The opportunity to engage with the content by completing exercises during class time, something strongly associated with flipped classrooms, seemed to resonate with individuals who liked being able to: '(…) ask questions and actually doing exercises in class' (student 1). Other participants shared appreciation for this aspect, but one student expressed that some more variety in content engagement would have been helpful: "I liked the emphasis on solving exercises, but I think I may have benefitted if there was more proving theorems and discussion fundamental notions in class" (student 9). This comment may have referred to a desire for more opportunities to observe the lecturer demonstrating these skills in the lessons, more than was offered in the course. For instance, more explicit demonstrations on the whiteboard and during lectures (e.g., students 4 & 9). This desire may have resulted from the high complexity and novelty of the mathematics within the course where students may require more guidance and explicit instruction. Likewise, it may be that the inherent complexity observed in a graduate-level course, along with the fast pace, 'Made it harder to quickly build understanding of the content as opposed to being able to actively engage with the lecturer as they explain the content' (student 2). This comment could be comparing the decrease in explicit instruction and the instructor's demonstration, which is more typically seen in traditional lectures.

Social Interaction

The final theme was that of *Social Interaction*. This encompassed codes about opportunities for interaction with peers and the instructor that were afforded by integrating flipped learning approaches in the lessons. This theme may be unsurprising, as the emphasis on collaborative practices was a noteworthy change for students compared to their other mostly traditional style courses. There is a notable variation in the codes within this theme, highlighting students' preferences and a need to consider these in any setting. A few students commented favourably about the engagement with peers and instructors in the course: 'I liked (…) the engagement with peers and with the lecturer' (student 5); however, most students did not explicitly discuss this in their responses. Regarding the learning atmosphere, one student described flipped learning as an environment conducive to asking questions; 'There are [sic] adequate time for ask [sic] questions' (student 1). On the other hand, the flipped nature may have incited more questions, which students felt could not be addressed or attended to. The following excerpt illustrates this:

> ‘Flipped lectures are ok, because they provide dynamic feedback and seem to foster peer collaboration, however I still have a tendency to get lost as the pace is still fast, and I am not able to clarify all my confusions (this would entail me asking a question every 10 seconds so would be unreasonable).’ (student 5)

When comparing the current course with other more traditional mathematics courses in the past, a similar view was expressed about the preference for traditional lectures in addressing clarifications over the current flipped format. Perhaps, similar to the previous student excerpt, the pace of the course, instructor availability, or another factor may have made it more challenging to engage in such forms of interactions:

> ‘I think the majority of the classes should still be taught in the traditional lecture format though as it is much easier to clarify something that you don't understand in this setting, as opposed to going through flipped lectures, making a list of what you don't understand and then having to e-mail the lecturer/attend office hours etc.’ (student 11)

On a similar note, student 4 states that in more traditional settings, ‘it just feels like you can ask questions as you're learning and not leave out any holes right at the beginning.’

Resources

The final theme encompasses codes directly linked to the resources students were provided with and their experience engaging with them outside of class. This theme is important to consider as one notable aspect of flipped learning is providing students with an adequate opportunity to appropriately engage with content in a way that best assumes the role of traditional lectures. This would include ensuring it is easy to use and navigate between resources.

One student notes the abundance of ‘all available resources (videos, flipped classes, notes, textbooks)’ (student 7) as a benefit. However, it is not hard to imagine that providing many resources can just as easily be experienced as a burden, especially if a student believes they lack the necessary background knowledge, making it hard to know how to use various resources. To mitigate this potential source of difficulty, student 6 suggested that it ‘Would've been better if we followed only one [set of videos] videos or [the textbook]’ (student 6). The diversity of resources resulted in inconsistencies in the definitions of key concepts: ‘The lectures often taught things different to [the textbook] which meant people had different definitions for things and it was difficult to reconcile these’. Using external resources is a defining feature of any flipped learning experience (Flipped Learning Network (FLN), 2014) and one that will depend on when students work with more complex mathematical concepts.

DISCUSSION

This study aimed to explore the flipped learning facet, an unfamiliar approach to learning mathematics for many students, introduced to a graduate-level mathematics course and how students perceived it. The thematic analysis identified four key themes that students reported, which closely align with many of the key characteristics of flipped classrooms. Students' experiences within this graduate-level course are parallel to similar studies, suggesting that there may be more commonalities between implementing flipped learning than we think. For instance, the emphasis on problem-solving in flipped learning and opportunities for collaboration with peers are commonly lauded (Lo et al., 2017). Additionally, a notable increase in student workload (in *Preparation and Workload)* is a widely reported consequence of realising flipped classrooms in the literature (Cevikbas & Kaiser, 2022), one which can result in students falling behind and being unable to participate.

The participants in this study showed a preference for traditional instructional modes, with some students finding it easier to seek clarifications and ask questions in lectures than in a flipped classroom. Moreover, the difficulty of the course and the demanding nature of the content may have also shaped this perception. Previous research suggests that an inclination for lectures may be due to the greater structure of lessons or the higher levels of reported concentration during them (Feudel & Fehlinger, 2023). Additionally, Novak et al. (2017) state that while flipped learning may be appropriate for the development of practical skills, the introduction of concepts may be better suited to lectures where a more knowledgeable figure can better support mathematics learning through explicit instruction (for review, see Evans & Dietrich, 2022). This is similar to mathematicians who often lean towards lectures as a preferred mode of learning from their colleagues as a means to support engagement with new mathematical areas (Weber & Fukawa-Connelly, 2023). Similarly, high-achieving students may share this sentiment for lectures as they encounter novel mathematics.

This study contributes to the graduate mathematics literature by providing one of the first studies to report on implementing flipped learning at this level. By doing so, we have shown that certain aspects can be well-received by students while others require further consideration. Implementing the main pillars of flipped learning may be more intricate at the graduate level, and this study paves the way for future research to explore such avenues. The limitations of a small-scale exploratory case study are that it is difficult to generalise our findings. The insights from this study contribute to the broader conversation on instructional approaches in graduate mathematics education and the student experiences shaped through them.

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