# New Zealand Mathematical Society Education Group

## Feedback on the Common Practice Model

The section on the pedagogical approach, *Thinking and working mathematically*, is excellent. It describes mathematics and mathematics learning close to the way mathematicians use mathematics and think about the subject. We want teachers and students to experience all the processes of mathematics (conjecturing, proving, generalising, etc.), which will help them make connections between the different areas and consolidate their teaching and learning (Knox & Kontorovich, 2023).

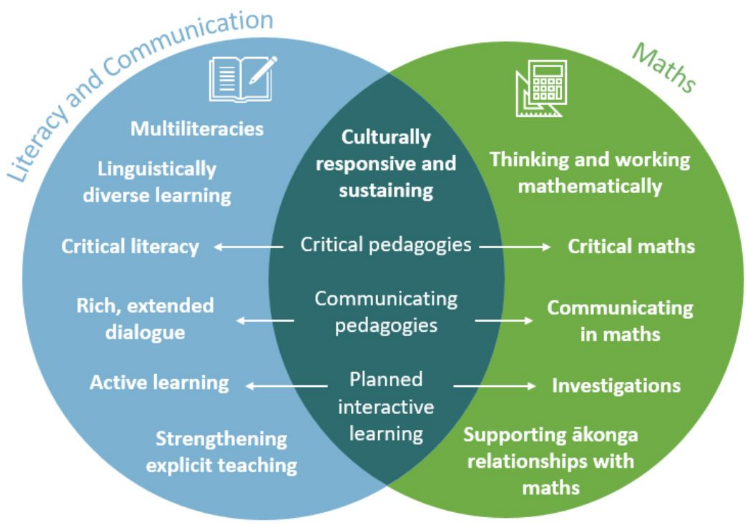
### Proposed Modifications

#### Critical Pedagogies

Within the *Critical pedagogies approach*, care must be taken to avoid mixing up mathematics with how it is applied. In the sentence, *Ākonga are encouraged to interrogate dominant discourses and assumptions, including that maths is benign, neutral, and culture-free*, one should avoid referring to mathematics itself and refer instead to the uses (and misuses) of maths. Although mathematics can only develop as part of human culture and society, at an abstract level, there is something in its nature that is independent of culture. The mathematical content of a statement is either *true* or *false*, and this feature remains unchanged whatever the context. It is not the case that culture and societal hierarchies influence the validity of mathematical statements. (E.g., even if the top mathematicians in a particular field (or country/culture) are convinced by some incorrect proof, it doesn't make it true, or vice versa.) However, students should develop a critical awareness of how maths is used and misused to justify certain policies and examine the assumptions that go into a mathematical model. Students should be encouraged to use mathematics as a tool for critical thinking and and to be aware of its vital role in society.

#### Explicit Teaching

*Strengthening explicit teaching* is mentioned as a pedagogical approach for Literacy and Communication. We strongly suggest it should also be stated for Mathematics and not taken for granted. In the Venn diagram of CPM pedagogies (copied below), explicit teaching should be moved from the left into the overlapping region.



Much of page 22 in the CPM document should be adapted for mathematics to ensure teachers are encouraged to provide expert explanations of new concepts and demonstrate worked examples before asking students to complete tasks. Mathematical concepts have been developed over the millennia that are built on each other and constitute a complex structure of abstraction. Students should not be expected to get very far by self-exploration. Expert guidance and quality explanations are needed, which is where the expert knowledge of a teacher plays a crucial role.

A large body of research from the science of learning has demonstrated the effectiveness of explicit teaching and debunked the assumption that students learn best when they are not provided expert explanations, such as in pure Inquiry-Based Learning (for review, see Evans & Dietrich, 2022; Kirschner et al., 2006; Sweller, 2021). Moreover, a much-touted 'productive failure' approach (Kapur & Bielaczyc, 2012) has not been replicated in the recently conducted randomised controlled trials published in a prestigious Educational Psychology Review (Ashman et al., 2020). It was long assumed (based on publications by Manu Kapur) that productive failure, where a problem-solving phase precedes explicit instruction, is more effective than explicit instruction followed by problem-solving. This prediction was tested with Year 5 primary school students learning about light energy efficiency. Two, fully randomised, controlled experiments were conducted. The first experiment ($N = 64$), demonstrated that explicit instruction followed by problem-solving was superior to the reverse order, measured by performance on problems similar to those used during instruction. No difference was found in performance on transfer problems (new problems with a similar structure set in different contexts). However, in the second experiment, where element interactivity was increased ($N = 71$), explicit instruction followed by problem-solving was found to be superior to the reverse order for performance on both similar and transfer problems. Most of the mathematical problem-solving with novel concepts involves high-element interactivity, according to the well-evidenced cognitive load theory and our current understanding of Human Cognitive Architecture (Sweller, 2008; Sweller et al., 2019).

Students benefit from explicit expert explanations as long as they are of high quality and engaging. It is our conviction that the pendulum of reforms to instil student-centred pedagogies has swung too far, thereby reducing the role of the teacher to a facilitator of learning with detrimental effects. The commonly used counter-argument that mathematics learning in NZ schools is mainly based on explicit teaching is not convincing as NZMS Education Group members are familiar with some primary, intermediate and secondary schools that strongly endorse Inquiry-Based Learning. Perhaps, there is a considerable variation in pedagogy used in NZ schools.

Good explicit teaching is critical for increasing representation in mathematics from traditionally underrepresented groups (Māori, Pasifika) and those of lower socioeconomic backgrounds who cannot rely on parents or private tutors to provide explicit teaching to fill in the gaps. However, we do not advocate for explicit teaching as a return to old-school approaches that might give teachers an excuse to go back to pushing content onto students without considering their needs. The best way to avoid that is to describe what good explicit teaching looks like in detail, complete with examples and specific advice. It needs to be mentioned as a pedagogical approach that is combined with culturally responsive approaches to best support learners (Averill et al., 2009; Hunter & Miller, 2022; Trinick & Meaney, 2017).

Although mathematics teaching is much more explicit at the NCEA level (in some but not all schools), there needs to be more at primary and intermediate levels across the board to give students a good grounding. In particular, expert quality explanations should be provided on basic arithmetic concepts and operations: addition, subtraction, multiplication and division algorithms. These are critical for basic numeracy.

**Example:** A teacher provides clear explanations on how to add numbers using column addition (with two or more digits) and takes time to explain why column addition works with any numbers. This involves careful monitoring of placevalue understanding by prompting learners to consider different scenarios and potential misconceptions. However, the teacher, when finding

52+48 = 100, may prompt students to notice a shortcut because the numbers are 50±2. Students might notice other patterns. They can share these observations with the teacher and fellow students. Students will open up in a culturally sustaining environment where communication is encouraged. If they miss these insights, they can still fall back on a method that always works - column addition.

## References


Ashman, G., Kalyuga, S., & Sweller, J. (2020). Problem-solving or Explicit Instruction: Which Should Go First When Element Interactivity Is High? *Educational Psychology Review*, 32(1), 229–247. https://doi.org/10.1007/s10648-019-09500-5

Averill, R., Anderson, D., Easton, H., Maro, P. T., Smith, D., & Hynds, A. (2009). Culturally Responsive Teaching of Mathematics: Three Models from Linked Studies. *Journal for Research in Mathematics Education*, 40(2), 157–186.

Evans, T., & Dietrich, H. (2022). Inquiry-based mathematics education: a call for reform in tertiary education seems unjustified. *STEM Education*, 2(3), 221–244. https://doi.org/10.3934/steme.2022014

Hunter, J., & Miller, J. (2022). Using a Culturally Responsive Approach to Develop Early Algebraic Reasoning with Young Diverse Learners. International Journal of Science and Mathematics Education, 20(1), 111-131. https://doi.org/10.1007/s10763-020-10135-0

Kapur, M., & Bielaczyc, K. (2012). Designing for productive failure. *Journal of the Learning Sciences*, 21(1), 45–83.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching. *Educational Psychologist*, 41(2), 75–86. https://doi.org/10.1207/s15326985ep4102_1

Knox, J., & Kontorovich, I. (2023). Leveraging interdiscursivity to support elementary students in bridging the empirical-deductive gap: the case of parity. *The Journal of Mathematical Behavior*, 70, 101052. https://doi.org/https://doi.org/10.1016/j.jmathb.2023.101052

Sweller, J. (2008). Human cognitive architecture. *Handbook of research on educational communications and technology*, 35, 369–381.

Sweller, J. (2021). Why Inquiry-based Approaches Harm Students' Learning. *In: The Centre for Independent Studies, Australia.*

Sweller, J., van Merriënboer, J. J., & Paas, F. (2019). Cognitive architecture and instructional design: 20 years later. *Educational Psychology Review*, 31(2), 261–292. https://doi.org/10.1007/s10648-019-09465-5

Trinick, T., & Meaney, T. (2017). Indigenous Teacher Education: When Cultural Enquiry Meets Statistical Enquiry. In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 514–521.) MERGA.