A modification of RCC*-9

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Abstract

RCC*-9 is a recently developed member of the region connection calculus family, and was introduced to represent topological relations in multidimensional space. In this paper, we discuss and address an inconsistency found in the RCC*-9 formalism, and propose a modified version of RCC*-9 which we call RCC*′-9. Furthermore, we prove the jointly exclusive and pairwise disjoint property of relations and theorems in the lattice of relations in the modified theory. Finally, we confirm the consistency of RCC*′-9 using a finite model finder.

Keywords: Multidimensional topological space, Region Connection Calculus, Qualitative spatial reasoning, Topological relationships.

1 Introduction

The formalization of topological relations has been an important research topic in the Geographical Information Systems (GIS) and Qualitative Spatial Reasoning (QSR) literature. These relations enable certain kinds of spatial querying, analysis and reasoning in GIS, such as whether two features are in connected and if so what type of connection exists between them (e.g. whether one feature is proper part of the other, or whether two features touch or overlap each other). Some examples of topological relations include: a polygon representing a national park overlaps with several adjacent polygons representing different countries; census blocks lie entirely within a census tract; parcels, have edges that touch each other.

There are two main approaches to modeling topological relations between spatial entities in the literature. The first of these follows an intersection strategy, and includes the 9-intersection model (9-IM) (Egenhofer and Herring, 1991) its dimensionally extended version (DE-9IM) (Clementini et al., 1994), and the calculus-based method (CBM) (Clementini et al., 1993). Their spatial features consist of points, line segments and areas, thereby describing spatial objects in terms of their dimension which is similar to geometric data standards, such as the Open GeoSpatial Consortium (OGC). 9-IM (and DE-9IM) considers each object's interior (^{o}), boundary (δ) and exterior (^{e}) parts and then represents the spatial relation between pairs of objects using a matrix in which rows

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represent the aspects (interior, boundary and exterior) of one object and columns represent the aspects of the other object, cell values being 0 or 1, indicating whether or not the object aspects intersect (dimension of the common part is also considered in DE-9IM). Because it considers every possible combination of these three aspects of an object, 9-IMs[1] introduce a large number of relations in a multidimensional space, and are thus not very user friendly. Also, it can only extract further knowledge over areal features. CBM reduces the number of relations by excluding the exterior aspect (i.e. $F1^o$-$F2^e$, $\delta F1$-$F2^e$, $F1^e$-$F2^e$ intersections are omitted)[2]. Instead, it describes the other intersections (interior and boundary) in terms of the participant objects (i.e., whether the common part is equal to one of the participants). Although it introduces a set of practical relations for human use, it does not support qualitative reasoning.

The second main approach adopts axiomatic systems such as (Clarke, 1981; Clark, 1985; Randell and Cohn, 1992) to represent topological relations. These systems are not only capable of introducing a set of relations for end users, but also supporting automated reasoning, i.e., inferring the existence of the relations to additional, unnamed entities, over all of the features in the spatial domain either by constructing a composition table based on the set of finite *jointly exclusive and pairwise disjoint* (JEPD) relations or directly through automated reasoners such as as first-order theorem provers or finite model finders. However, with few exceptions (Gotts work (Gotts, 1996), Galton's work (Galton, 1996) and CODI (Hahmann and Grüninger, 2011; Hahmann, 2018)) they only accept equidimensional spatial entities (e.g., only areal features) in their domain. For instance, equidimensional axiomatic systems are not able to describe a topological relationship between a road and park, if these objects are represented as a one-dimensional and two-dimensional spatial features, respectively, in a geographical dataset.

The development of a comprehensive theory of topological relations to overcome the respective shortcomings of the intersection approach (a large number of relations without a practical reasoning strategy) and axiomatic systems (inability to handle objects of different dimensions in the spatial domain) has long been an open problem in GISience (Galton, 2004). Much existing work (Gotts, 1996; Galton, 1996; Hahmann and Grüninger, 2011; Hahmann, 2018) on extending the axiomatic approach to a truly multidimensional theory has focused on developing a first-order logical axiomatization that affords reasoning with theorem provers. This work studies an alternative approach, the RCC*-9 (Clementini and Cohn, 2014), which more closely follows the early work in qualitative spatial reasoning by aiming to identify a set of JEPD relations to support composition based reasoning with the help of a composition table, which allows simply looking up the results of combinations of spatial relations. A second notable difference is that unlike (Gotts, 1996; Galton, 1996; Hahmann, 2013), (Clementini and Cohn, 2014) does not include a predicate (or predicates) for comparing the dimension of the participant entities. (Hahmann, 2018) includes a primitive relation of "lesser or equal dimension" and (Gotts, 1996; Galton, 1996) define similar relations in their formalism. RCC*-9 (Clementini and Cohn, 2014) aims to define a multidimensional theory without any dimensional comparison tool. However, a closer analysis of the relations in RCC*-9 reveals that they do not represent the expected spatial configuration, which means that the relations are not JEPD and therefore do not satisfy the lattice of the relations. In this paper, we modify the treatment of topological relations among multidimensional features introduced in RCC*-9 to address this problem.

This paper is structured as follows. In Section 2, we explain RCC-8 and RCC*-9 as homogeneous

[1] Although the earlier version of 9-IM (Egenhofer and Franzosa, 1991) is not for multidimensional cases, it follows the same strategy. However, it uses rules to eliminate unnecessary combinations.

[2] $F1$ and $F2$ are two sample spatial features.

and heterogeneous dimensional theories in the region connection calculus framework, and distinguish their attributes and capabilities. In Section 3, we focus on RCC*-9's formalism to explain its weaknesses and propose a solution. Theorems of JEPD properties and a lattice of relations are also proved over the solution. Finally, we check the consistency of the whole formalism in Section 4. Possibilities for extending this work are discussed in Section 5.

2 Region Connection Calculi (RCC)

RCC is a family of qualitative representations of topological relations between regions, and is used for constraint-based qualitative spatial reasoning. It has been extended in various directions, and in the following sections, we review one of the most well-known members of this family, RCC-8, and a more recent multidimensional version, RCC*-9, which is the focus of the current work.

2.1 RCC-8

RCC-8 (Randell et al., 1992) proposed a point-free topological space. Entities of the domain are known as regions in this theory. The regions are (non-empty) chunks of space occupied by physical objects. There is no dimensional difference between the regions and the universal embedding space (zero co-dimension). Moreover, every region only consists of equi-dimensional parts (regular subsets of the space). Also, there is no requirement for the regions to be internally connected (multi-piece regions are permitted).

RCC-8 is based on a single primitive binary relation: **C**(x,y), read as 'x connects with y', which is reflexive and symmetric. This relation holds when there is an overlap between the closures[3] of x and y, that is, when $\mathrm{cl}(x) \cap \mathrm{cl}(y) \neq \varnothing$.

Based on the **C** relation, the additional topological relations shown in Figure 1 are defined. Among them, a set of eight relations (numbered relations in Figure 1) form jointly exhaustive and pairwise disjoint (JEPD) set of relations, called *base relations*. It means that each pair of spatial regions of the considered domain is in exactly one of the eight JEPD relations. Since the spatial domain is indefinite, reasoning techniques mostly rely on verified composition of two base relations. So, an 8 by 8 composition table of base relations has been constructed for RCC-8.

Furthermore, a set of Boolean operations, including `sum`, `product`, `difference`, and `complement` are defined in the logical axiomatization of RCC. The domain is closed under these operations, as guaranteed by the introduction of the *NULL* entity, being defined as the `product` of the discrete regions in the domain. Likewise, all the regions are connected to a specific region known as the *universal* region which is an upper bound of the domain.

2.2 RCC*-9

RCC*-9 (Clementini and Cohn, 2014) extends RCC-8 by admitting the coexistence of regions of heterogeneous dimensions[4] Indeed, the regions are not lumps of space filled by physical objects as

[3]In point-set topology, the closure of a subset S of points in a topological space consists of all points in S together with all limit points of S.

[4]The reason for the asterisk in the name of RCC*-9 does not only indicate a change in the number of relations in comparison to RCC-8, there is also an additional spatial primitive that the new calculus is able to deal with.

1. $\mathbf{DC}(x,y) \equiv_{def} \neg\mathbf{C}(x,y)$ (x disconnected from y)
   $\mathbf{P}(x,y) \equiv_{def} \forall z[\mathbf{C}(z,x) \rightarrow \mathbf{C}(z,y)]$ (x is part of y)
   $\mathbf{PP}(x,y) \equiv_{def} \mathbf{P}(x,y) \wedge \neg\mathbf{P}(y,x)$ (x is proper part of y)
2. $\mathbf{EQ}(x,y) \equiv_{def} \mathbf{P}(x,y) \wedge \mathbf{P}(y,x)$ (x equals to y)
   $\mathbf{O}(x,y) \equiv_{def} \exists z[\mathbf{P}(z,x) \wedge \mathbf{P}(z,y)]$ (x overlaps y)
   $\mathbf{DR}(x,y) \equiv_{def} \neg\mathbf{O}(x,y)$ (x is discrete from y)
3. $\mathbf{NTPP} \equiv_{def} \mathbf{PP}(x,y) \wedge \neg\exists z[\mathbf{EC}(z,x) \wedge \mathbf{EC}(z,y)]$ (x is non-tangential proper part of y)
4. $\mathbf{TPP} \equiv_{def} \mathbf{PP}(x,y) \wedge \exists z[\mathbf{EC}(z,x) \wedge \mathbf{EC}(z,y)]$ (x is tangential proper part of y)
5. $\mathbf{PO}(x,y) \equiv_{def} \mathbf{O}(x,y) \wedge \neg\mathbf{P}(x,y) \wedge \neg\mathbf{P}(y,x)$ (x partially overlaps y)
6. $\mathbf{EC}(x,y) \equiv_{def} \mathbf{C}(x,y) \wedge \neg\mathbf{O}(x,y)]$ (x externally connected to y)
   $\mathbf{Pi}(x,y) \equiv_{def} \mathbf{P}(y,x)$ (y has part x)
   $\mathbf{PPi}(x,y) \equiv_{def} \mathbf{PP}(y,x)$ (y has proper part x)
7. $\mathbf{TPPi}(x,y) \equiv_{def} \mathbf{TPP}(y,x)$ (y has tangential proper part x)
8. $\mathbf{NTPPi}(x,y) \equiv_{def} \mathbf{NTPP}(y,x)$ (y has non-tangential proper part x)

Figure 1: Defined relations in the RCC-8 from Randell and Cohn (1992)

they are in RCC-8. They are represented based on the terminology of the features in the OGC (OGC, 2010), and can be points, linear or areal features. It is assumed that linear features are topologically closed (i.e. bounded by two, possibly coincident, endpoints), and that areal features are regularly closed (i.e. bounded by a single or multiple linear regions). To follow the OGC standard, objects with holes or multiple parts must also be supported.

The theory not only has the **C** relation as a primitive, but also utilizes a second primitive relation, **B**(x,y), read as 'x is boundary of y', such that x must be a proper part of y (i.e. ∀x∀y **B**(x,y) → **PP**(x,y) is an axiom). So, the boundary of an areal feature is its limiting closed curve, and the set of endpoints are considered the boundary of a linear feature. A point (or set of points) does not have any boundary.

Based on these primitive relations, the set of spatial relations shown in Figure 3 are defined. The definitions of **DC**, **P**, **PP**, and **EQ** are preserved from the RCC-8. However, the introduction of the new primitive relation causes some alteration in the definitions of other relations in RCC-8 such as **DR**, **NTPP**, **TPP**, **NTPPi**, **TPPi**, **O**, **PO**, and **EC**. Moreover, the boundary relation facilitates the introduction of a new spatial relation, **CR**(x,y), read as 'x crosses y', which can only hold between two linear features. The nine numbered relations in Figure 3 have again the JEPD property. Clementini and Cohn (2014) provide a composition table for RCC*-9 (see Figure 4 on p.14), and RCC*-9's composition table has an extra row and column relative to the RCC-8 composition table, corresponding to the **CR** relation. Also, whenever the composition of the two relations returns an overlap relation (or its special case, **PO**) in RCC-8's table, there is a possibility of seeing the **CR** relation between the participants in the entry table as well.

In short, RCC*-9 differs mostly from RCC-8 by accepting entities of different dimensions. It is a boundary-tolerant theory, in contrast to RCC-8, and is based on two primitives, **C** & **B**. The introduction of a new base relation, **CR** in RCC*-9 increases its expressiveness. Since RCC*-9 is based on the OGC's definitions of features (OGC, 2010), it may be considered more applicable in the geographic domain than RCC-8. A summary of the differences is shown in Table 1.

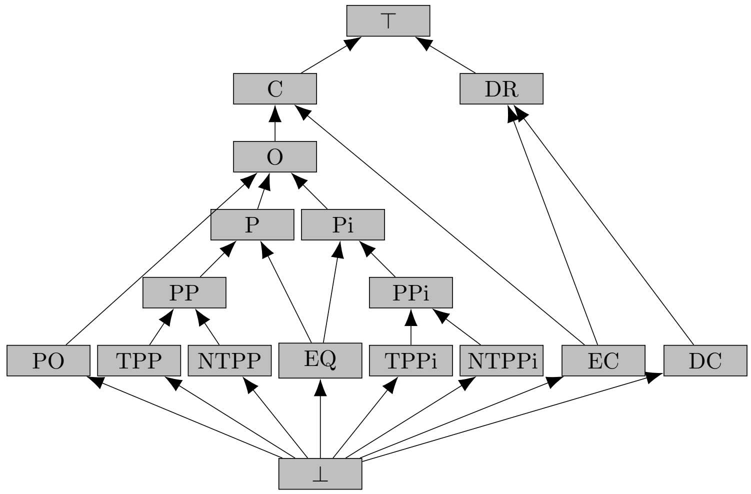

Figure 2: The lattice of RCC-8 topological relations from Randell and Cohn (1992). $\top$ denotes the universal relation that applies to any pair of regions (i.e., true) and $\bot$ denotes the empty relation that never holds (i.e., is always false). Arrows denote a specialization, e.g., the arrow from **EC** to **DR** shows that **EC** specializes **DR**, i.e. if $\mathbf{EC}(x, y)$ is true for arbitrary x and y then $\mathbf{DR}(x, y)$ must also be true. The lattice consists of jointly exhaustive relation. For example, if $\mathbf{DR}(x, y)$ holds for arbitrary x asn y, then either $\mathbf{EC}(x, y)$ or $\mathbf{DC}(x, y)$ is implied. The relations are pairwise disjoint, for example, $\mathbf{EC}(x, y)$ and $\mathbf{DC}(x, y)$ cannot be true at the same time for a pair x and y.

Table 1: Comparison of RCC-8 and RCC*-9

Theory / Properties	RCC-8	RCC*-9
Entities	Homogeneous regions Regular closed	Heterogeneous regions Regular closed
Boundary	Without	With
Primitive relation(s)	1	2
Number of base relations	8	9
Boolean operators	Yes	No
Composition table	Available(8*8)	Available(9*9)
Application	All physical processes	Geographical domain

3 RCC*-9 under surveillance

When we study RCC*-9 in more detail, its lattice of relations (Figure 4) gives rise to some logical statements that must be true in the logical axiomatization of RCC*-9, in order for the lattice to be correct. These statements can be divided into two types: *specialization* and *subsumption*. Specialization is captured by rule I, while rule II captures subsumption.

I) Where there is an edge between two relations in a lattice, some source relation **S** (lower in the lattice) implies the target relation **T** (further up in the lattice):

$$\mathbf{S}(x, y) \rightarrow \mathbf{T}(x, y)$$

For example, $\mathbf{CR}(x, y) \rightarrow \mathbf{C}(x, y)$.

When one relation points to (i.e. specializes) more than a single relation (e.g., **EC** specializes **C** and **DR**), then the specialized relation implies all of the relations it points to. For example, $\mathbf{EC}(x, y) \rightarrow \mathbf{DR}(x, y) \wedge \mathbf{C}(x, y)$.

1. $\mathbf{DC}(x,y) \equiv_{def} \neg\mathbf{C}(x,y)$ (x disconnected from y)
   $\mathbf{P}(x,y) \equiv_{def} \forall z[\mathbf{C}(z,x) \rightarrow \mathbf{C}(z,y)]$ (x is part of y)
   $\mathbf{PP}(x,y) \equiv_{def} \mathbf{P}(x,y) \wedge \neg\mathbf{P}(y,x)$ (x is proper part of y)
2. $\mathbf{EQ}(x,y) \equiv_{def} \mathbf{P}(x,y) \wedge \mathbf{P}(y,x)$ (x equals to y)
3. $\mathbf{NTPP}(x,y) \equiv_{def} \mathbf{PP}(x,y) \wedge \forall z[\mathbf{B}(z,y) \rightarrow \mathbf{DC}(x,z)]$ (x is non-tangential proper part of y)
4. $\mathbf{TPP}(x,y) \equiv_{def} \mathbf{PP}(x,y) \wedge \neg\mathbf{NTPP}(x,y)$ (x is tangential proper part of y)
   $\mathbf{O}(x,y) \equiv_{def} \exists z[\mathbf{NTPP}(z,x) \wedge \mathbf{NTPP}(z,y)] \wedge \exists t[\mathbf{TPP}(t,x) \wedge \mathbf{TPP}(t,y)]$ (x overlaps y)
5. $\mathbf{PO}(x,y) \equiv_{def} \mathbf{O}(x,y) \wedge \neg\mathbf{P}(x,y) \wedge \neg\mathbf{P}(y,x)$ (x partially overlaps y)
6. $\mathbf{EC}(x,y) \equiv_{def} \mathbf{C}(x,y) \wedge \neg\mathbf{O}(x,y) \wedge \forall z[[\mathbf{P}(z,x) \wedge \mathbf{P}(z,y)] \rightarrow \mathbf{TPP}(z,x) \vee \mathbf{TPP}(z,y)]$($x$ externally connected to y)
   $\mathbf{DR}(x,y) \equiv_{def} \mathbf{EC}(x,x) \vee \mathbf{DC}(x,y)$ (x is discrete from y)
7. $\mathbf{CR}(x,y) \equiv_{def} \mathbf{C}(x,y) \wedge \neg\mathbf{O}(x,y) \wedge \neg\mathbf{EC}(x,y)$ (x crosses y)
   $\mathbf{Pi}(x,y) \equiv_{def} \mathbf{P}(y,x)$ (y has part x)
   $\mathbf{PPi}(x,y) \equiv_{def} \mathbf{PP}(y,x)$ (y has proper part x)
8. $\mathbf{TPPi}(x,y) \equiv_{def} \mathbf{TPP}(y,x)$ (y has tangential proper part x)
9. $\mathbf{NTPPi}(x,y) \equiv_{def} \mathbf{NTPP}(y,x)$ (y has non-tangential proper part x)

Figure 3: Defined relations in the RCC*-9 from Clementini and Cohn (2014).

II) Where two (or more) relations $\mathbf{S}_1$ to $\mathbf{S}_n$ specialize a single relation $\mathbf{T}$ (e.g. **CR**, **O** and **EC** all specialize **C**), then the disjunction of the specialized relations is equivalent to the target relation:

$$\mathbf{T}(x,y) \leftrightarrow \mathbf{S}_1(x,y) \vee \cdots \vee \mathbf{S}_n(x,y)$$

Also, the nine base relations, $\mathbf{R}_i$, of RCC*-9 ($\mathbf{R}_i\ i : 1,2,...,9$) must satisfy the following two properties as well:

III) $\neg\mathbf{R}_{i1}(x,y) \vee \neg\mathbf{R}_{i2}(x,y)$ (pairwise disjoint),

IV) $\mathbf{R}_1(x,y) \vee \mathbf{R}_2(x,y) \vee \ ... \vee \mathbf{R}_9(x,y)$ (jointly exhaustive).

To verify the lattice of RCC*-9, we applied and checked these rules (I - IV) over the relations. However, we identified some problems. According to the above mentioned properties in the lattice of RCC*-9, the overlap relation is a generalized form of the **PO**, **P** and **Pi** relations, so according

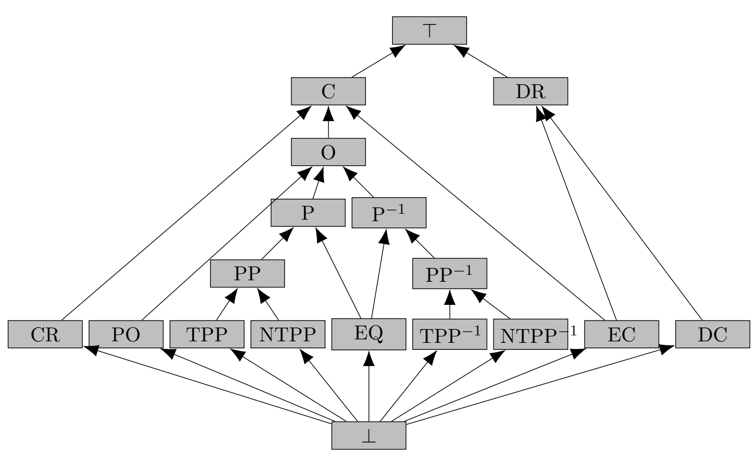

Figure 4: The lattice of the RCC*-9's spatial relations from Clementini and Cohn (2014).

to rule II, we have:

$$\mathbf{O}(x,y) \leftrightarrow \mathbf{PO}(x,y) \vee \mathbf{P}(x,y) \vee \mathbf{Pi}(x,y) \quad (A)$$

This predicate means that not only must the overlap relation (**O**(x,y)) imply its specialized relations, the specialized relations must also imply the overlap relations. In other words, the overlap relation must cover all of thee subsumed relations directly (**P**,**Pi** and **PO**) and indirectly (**PP**,**PPi** ,**NTPP**, **NTPPi**, **TPP** and **TPPi**).

On the other hand, the definition of the **O**(x,y) from Figure 3 says:

$$\exists z[\mathbf{NTPP}(z,x) \wedge \mathbf{NTPP}(z,y)] \wedge \exists t[\mathbf{TPP}(t,x) \wedge \mathbf{TPP}(t,y)]$$

By considering (A), we expect that **O**(x,y) is entailed by **P**(x,y):

$$\mathbf{P}(x,y) \rightarrow \mathbf{O}(x,y) \quad (B)$$

Since we fail in showing that the union of a set of all axioms and defined relations (Γ), and the negation of (B) is not satisfiable, there must be a model for it. The finite model finder, Mace4 (McCune, 2006), searches for finite models of it. For a given size two domain ({**0**,**1**}), all instances of the union over this domain are generated (see Table 2). As you can see, while there is a model for **P**(x,y) (i.e.,"1"s in P: table), there is not any model for **O**(x,y) (i.e., "0"s in O: table). Alternatively, the model is a counter-model for $\Gamma \cup (\mathbf{P}(x,y) \rightarrow \mathbf{O}(x,y))$, and so **P**(x,y) does not entail **O**(x,y).

Table 2: Model provided by Mace4 for $\Gamma \cup \neg(\mathbf{P}(x,y) \rightarrow \mathbf{O}(x,y))$.

B:	**0**	**1**
0	0	0
1	0	0

C:	**0**	**1**
0	1	1
1	1	1

DC:	**0**	**1**
0	0	0
1	0	0

NTPP:	**0**	**1**
0	1	1
1	1	1

O:	**0**	**1**
0	0	0
1	0	0

P:	**0**	**1**
0	1	1
1	1	1

PP:	**0**	**1**
0	1	1
1	1	1

TPP:	**0**	**1**
0	0	0
1	0	0

The source of this unexpected behavior seems to be the definition of **O**(x,y). Its truth depends on the existence of an object (t) that is the tangential proper part of x and y at the same time. Such a t must be found in 'all' the specialized relations of **O**. However, if x is a non-tangential proper part of y, there is no such t (see Figure 5). So, RCC*-9 has defined a different notion of overlap than RCC-8. RCC*-9's overlap relation actually captures the partially overlap relation (see Figure

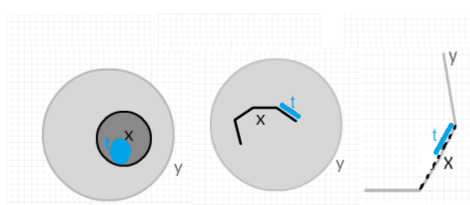

Figure 5: NTPP relation in RCC*-9

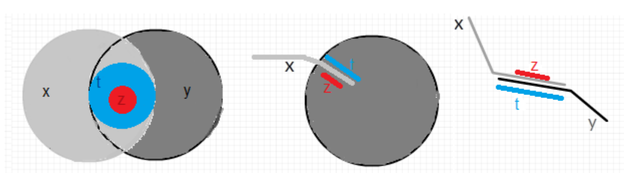

Figure 6: PO relation in RCC*-9

6) rather than overlap. In order to reflect this, we rename **O**(x,y) to partially overlap relation with prime, $\mathbf{PO'}$ (all the relations in our new theory use $'$ [prime] to distinguish them from the original versions):

$$\mathbf{PO'}(x,y) \equiv_{def} \exists z[\mathbf{NTPP}(z,x) \wedge \mathbf{NTPP}(z,y)] \wedge \exists t[\mathbf{TPP}(t,x) \wedge \mathbf{TPP}(t,y)]$$

and we redefine the overlap relation, $\mathbf{O'}(x,y)$, as follows:

$$\mathbf{O'}(x,y) \equiv_{def} \mathbf{PO'}(x,y) \vee \mathbf{P}(x,y) \vee \mathbf{Pi}(x,y)$$

Thus **O**(x,y) is replaced by $\mathbf{O'}$(x,y) in all the relations and theorems, so the definitions of $\mathbf{EC'}$(x,y) and $\mathbf{CR'}$(x,y) relations are modified consequently:

$$\mathbf{EC'}(x,y) \equiv_{def} \mathbf{C}(x,y) \wedge \neg\mathbf{O'}(x,y) \wedge \forall z[[\mathbf{P}(z,x) \wedge \mathbf{P}(z,y)] \rightarrow \mathbf{TPP}(z,x) \vee \mathbf{TPP}(z,y)]$$

$$\mathbf{CR'}(x,y) \equiv_{def} \mathbf{C}(x,y) \wedge \neg\mathbf{O'}(x,y) \wedge \neg\mathbf{EC'}(x,y)$$

We name this modified version of the theory RCC*′-9. The next step is to check the rules of the lattice over this modified set of relations. To do it, we check rules I and II on all of the relations. These rules construct a set of theorems that are listed and proved in Appendix A.

Moreover, the problem in the definition of overlap that has previously been mentioned has the result that overlap does not satisfy rule III over its subset relations. For instance, $\neg\mathbf{PO}(x,y) \vee \neg\mathbf{TPP}(x,y)$ is not provable, since partially overlap and proper part relations (by their definitions) do not represent completely distinguished spatial arrangements. Since the relations must have the JEPD property in order to support reasoning, we must also confirm that the modified set of relations are also JEPD. We achieve this by proving all the theorems shown in Appendix B, which are generated by applying rules III and IV.

Further clarification of the theory is also necessary to provide a clear description of the topological domain. To achieve this goal, more theorems are needed to put more restrictions on the RCC′*-9 relations, and these are contained in Appendix C (all references beginning with Ext.T in Table 3 and the following paragraphs refer to Appendix C). Specifically, the axioms of the theory imply some properties for relations as can be seen in Table 3. Here, the identity of two features is a special case of their equality (Ext.T.9), consequently two non-identical entities are not equal (Ext.T.10). Also, we conclude that a boundary part of a feature is its tangential proper part as well (Ext.T.29).

Table 3: Properties of the relations in the RCC*′-9 and their relevant theorems in Appendix C

Relation	**Properies**
DC(x,y)	Irreflexive (Ext.T.1), Symmetric (Ext.T.2)
P(x,y)	Reflexive (Ext.T.3), Anti-symmetric (Ext.T.4), Transitive (Ext.T.5).
EQ(x,y)	Reflexive (Ext.T.6), Symmetric (Ext.T.7), Transitive (Ext.T.8)
PP(x,y)	Irreflexive (Ext.T.11), Asymmetric (Ext.T.12), Transitive (Ext.T.13)
O′(x,y)	Reflexive (Ext.T.14), Symmetric (Ext.T.15)
DR(x,y)	Irreflexive (Ext.T.16), Symmetric (Ext.T.17)
PO′(x,y)	Irreflexive (Ext.T.18), Symmetric (Ext.T.19)
EC′(x,y)	Irreflexive (Ext.T.20), Symmetric (Ext.T.21)
TPP(x,y)	Irreflexive (Ext.T.22), Asymmetric (Ext.T.23)
NTPP(x,y)	Irreflexive (Ext.T.24), Asymmetric (Ext.T.25), Transitive (Ext.T.26)
CR′(x,y)	Irreflexive (Ext.T.27), Symmetric (Ext.T.28)

Theorems (Ext.T.30) and (Ext.T.31) show the relationship between the RCC*′-9 and the classical mereotopological calculus (Leonard and Goodman, 1940). With the absence of the cross relation in the set of supported relations, RCC*′-9 collapses to RCC-8. Finally, the theorem (Ext.T.32) says that a non-tangential proper part of a region is part of its interior, so the connection of a feature with any interior part of another feature implies either overlap or cross. Any part of the non-tangential proper part of a region is its non tangential proper part (Ext.T.33). If an entity is both part of a second entity and connected to a third entity, the two other (second and third) entities are connected (Ext.T.34).

4 Logical verification

To check the correctness of RCC*′-9, we exploit consistency checking, which is a standard technique in first order logic. It confirms that the formalism does not entail any contradiction after instantiating all the axioms and a set of provable theorems ($< F >$) over the domain. Mathematically, there must be no formula (ϕ) such that ϕ and $\neg\phi$ are a member of $< F >$ simultaneously.

This technique involves generating some finite models via a finite model finder. This technique is implemented in the Macleod suit of tools [5] that was previoulsy used to check the consistency of RCC-8 and some other theories with the help of the finite model finder Mace4. We used the same approach to prove the consistency of RCC*′-9.

5 Conclusion and further work

Since spatial features may be point, linear or areal features in GIS, having a qualitative theory that can support querying over multidimensional data is crucial. RCC*-9 aims to meet this goal. However, we demonstrate that the **O** relation in RCC*-9 does not capture the intended spatial configuration. The main contribution of this paper is the introduction of RCC*′-9 as a modification of RCC*-9 that resolves the identified problem. We prove the JEPD properties of the relations and theorems relevant to the lattice of relations in RCC*′-9 and we evaluate the consistency of the theory by finding finite models via Mace4.

Further research is needed to check whether the composition table of RCC*′-9 remains unchanged and also provide a formal proof of its correctness. In addition, verifying the theory with some sample data sets would be a further useful verification technique to further ensure the consistency and appropriateness of the model in real world scenarios.

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⋄	**DC**	**EC**	**PO**	**TPP**	**NTPP**	**TPPi**	**NTPPi**	**EQ**	**CR**
DC	*	DR, PO, PP, CR	DR, PO, PP, CR	DR, PO, PP, CR	DR, PO, PP, CR	DC	DC	DC	DR, PO, PP, CR
EC	DR, PO, PPi, CR	DR, PO,TPP, TPPi, EQ, CR	DR, PO, PP, CR	EC, PO, PP, CR	PO, PP, CR	DR	DC	EC	DR, PO, PP, CR
PO	DR, PO, PPi, CR	DR, PO, PPi, CR	*	PO, PP, CR	PO,PP, CR	DR, PO, PPi, CR	DR, PO, PPi, CR	PO	DR, PO, PP, PPi, CR
TPP	DC	DR	DR, PO, PP, CR	PP	NTPP	DR, PO, TPP, TPPi, EQ, CR	DR, PO PPi, CR	TPP	DR, PO, PP, CR
NTPP	DC	DC	DR, PO, PP, CR	NTPP	NTPP	DR, PO, PP, CR	*	NTPP	DR, PO, PP, CR
TPPi	DR, PO, PPi, CR	EC,PO PPi, CR	PO, PPi CR	PO, EQ, TPP, TPPi	PO, PP, CR	PPi	NTPPi	TPPi	PO, PPi, CR
NTPPi	DR, PO, PPi, CR	PO, PPi, CR		PO, PPi, CR	PO, PPi, CR	O, CR NTPPi	NTPPi	NTPPi	PO, PPi, CR
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ	CR
CR	DR, PO, PPi, CR	DR, PO, PPi, CR	DR, PO, PPi,PP, CR	PO, PP, CR	PO, PP, CR	DR, PO, PPi, CR	DR, PO, PPi, CR	CR	*

Table 4: Composition table of relations in RCC*-9

Appendices

A Theorems of lattice of relations in RCC*′9

A diagrammatic representation of the lattice is contained in (Figure.4) [6]. Here we provide a set of proved theorems. All these theorems are provable by using the refutation technique on the CNFs of the formulas.

$(T.1)$ $\forall x \forall y C(x,y) \vee DR(x,y)$
$(T.2)$ $\forall x \forall y C(x,y) \leftrightarrow CR'(x,y) \vee O'(x,y) \vee EC'(x,y)$
$(T.3)$ $\forall x \forall y DR(xy) \leftrightarrow EC'(x,y) \vee DC(x,y))$
$(T.4)$ $\forall x \forall y DC(x,y) \rightarrow DR(x,y)$
$(T.5)$ $\forall x \forall y EC'(x,y) \rightarrow C(x,y)$
$(T.6)$ $\forall x \forall y EC'(x,y) \rightarrow DR(x,y)$
$(T.7)$ $\forall x \forall y EC'(x,y) \rightarrow C(x,y) \wedge DR(x,y)$
$(T.8)$ $\forall x \forall y O'(x,y) \rightarrow C(x,y)$
$(T.9)$ $\forall x \forall y O'(x,y) \leftrightarrow P(x,y) \vee Pi(x,y) \vee PO'(x,y)$
$(T.10)$ $\forall x \forall y Pi(x,y) \rightarrow O'(x,y)$
$(T.11)$ $\forall x \forall y PPi(x,y) \rightarrow Pi(x,y)$
$(T.12)$ $\forall x \forall y PPi(x,y) \leftrightarrow TPPi(x,y) \vee NTPPi(x,y)$
$(T.13)$ $\forall x \forall y TPPi(x,y) \rightarrow PPi(x,y)$
$(T.14)$ $\forall x \forall y NTPPi(x,y) \rightarrow PPi(x,y)$
$(T.15)$ $\forall x \forall y EQ(x,y) \rightarrow Pi(x,y)$
$(T.16)$ $\forall x \forall y EQ(x,y) \rightarrow P(x,y)$
$(T.17)$ $\forall x \forall y P(x,y) \rightarrow O'(x,y)$
$(T.18)$ $\forall x \forall y PP(x,y) \rightarrow P(x,y)$
$(T.19)$ $\forall x \forall y PP(x,y) \leftrightarrow TPP(x,y) \vee NTPP(x,y)$
$(T.20)$ $\forall x \forall y TPP(x,y) \rightarrow PP(x,y)$
$(T.21)$ $\forall x \forall y NTPP(x,y) \rightarrow PP(x,y)$
$(T.22)$ $\forall x \forall y PO'(x,y) \rightarrow O'(x,y)$
$(T.23)$ $\forall x \forall y CR'(x,y) \rightarrow C(x,y)$

B Theorems of JEPD property of the RCC*′9

Below are assembled together a set of theorems that define the JEPD property of the RCC*′-9's relations. All these theorems are provable by using the refutation technique on the CNFs of the formulas.

$(T.24)$ $\forall x \forall y \neg CR'(x,y) \vee \neg DC(x,y)$
$(T.25)$ $\forall x \forall y \neg CR'(x,y) \vee \neg EC'(x,y)$
$(T.26)$ $\forall x \forall y \neg CR'(x,y) \vee \neg NTPPi(x,y)$
$(T.27)$ $\forall x \forall y \neg CR(x,y) \vee \neg TPPi(x,y)$
$(T.28)$ $\forall x \forall y \neg CR'(x,y) \vee \neg EQ(x,y)$
$(T.29)$ $\forall x \forall y \neg CR'(x,y) \vee \neg NTPP(x,y)$
$(T.30)$ $\forall x \forall y \neg CR(x,y) \vee \neg TPP(x,y)$

[6]These theorems are for RCC*′-9

$(T.31)$ $\forall x \forall y \neg CR(x, y) \vee \neg PO(x, y)$
$(T.32)$ $\forall x \forall y \neg PO(x, y) \vee \neg DC(x, y)$
$(T.33)$ $\forall x \forall y \neg PO(x, y) \vee \neg EC(x, y)$
$(T.34)$ $\forall x \forall y \neg PO(x, y) \vee \neg NTPPi(x, y)$
$(T.35)$ $\forall x \forall y \neg PO(x, y) \vee \neg TPPi(x, y)$
$(T.36)$ $\forall x \forall y \neg PO(x, y) \vee \neg EQ(x, y)$
$(T.37)$ $\forall x \forall y \neg PO(x, y) \vee \neg NTPP(x, y)$
$(T.38)$ $\forall x \forall y \neg PO(x, y) \vee \neg TPP(x, y)$
$(T.39)$ $\forall x \forall y \neg TPP(x, y) \vee \neg DC(x, y)$
$(T.40)$ $\forall x \forall y \neg TPP(x, y) \vee \neg EC(x, y)$
$(T.41)$ $\forall x \forall y \neg TPP(x, y) \vee \neg NTPPi(x, y)$
$(T.42)$ $\forall x \forall y \neg TPP(x, y) \vee \neg TPPi(x, y)$
$(T.43)$ $\forall x \forall y \neg TPP(x, y) \vee \neg EQ(x, y)$
$(T.44)$ $\forall x \forall y \neg TPP(x, y) \vee \neg NTPP(x, y)$
$(T.45)$ $\forall x \forall y \neg NTPP(x, y) \vee \neg DC(x, y)$
$(T.46)$ $\forall x \forall y \neg NTPP(x, y) \vee \neg EC(x, y)$
$(T.47)$ $\forall x \forall y \neg NTPP(x, y) \vee \neg NTPPi(x, y)$
$(T.48)$ $\forall x \forall y \neg NTPP(x, y) \vee \neg TPPi(x, y)$
$(T.49)$ $\forall x \forall y \neg NTPP(x, y) \vee \neg EQ(x, y)$
$(T.50)$ $\forall x \forall y \neg EQ(x, y) \vee \neg TPPi(x, y)$
$(T.51)$ $\forall x \forall y \neg EQ(x, y) \vee \neg NTPPi(x, y)$
$(T.52)$ $\forall x \forall y \neg EQ(x, y) \vee \neg EC(x, y)$
$(T.53)$ $\forall x \forall y \neg EQ(x, y) \vee \neg DC(x, y)$
$(T.54)$ $\forall x \forall y \neg TPPi(x, y) \vee \neg NTPPi(x, y)$
$(T.55)$ $\forall x \forall y \neg TPPi(x, y) \vee \neg EC(x, y)$
$(T.56)$ $\forall x \forall y \neg TPPi(x, y) \vee \neg DC(x, y)$
$(T.57)$ $\forall x \forall y \neg EC'(x, y) \vee \neg DC(x, y)$
$(T.58)$ $\forall x \forall y \neg EC'(x, y) \vee \neg NTPPi(x, y)$
$(T.59)$ $\forall x \forall y CR'(x, y) \vee PO'(x, y) \vee NTPP(x, y) \vee TPP(x, y) \vee EQ(x, y) \vee TPP^{-1}(x, y) \vee NTPP^{-1}(x, y) \vee EC'(x, y) \vee DC(x, y)$

C Other necessary theorems

Below are the set of theorems that are necessary to support the defined spatial relations in RCC$^{*\prime}$-9.

$(Ext.T.1)$ $\forall x \neg DC(x, x)$ $(From\ reflexivity\ of\ C\ and\ definition\ of\ DC)$
$(Ext.T.2)$ $\forall x \forall y DC(x, y) \rightarrow DC(y, x)$ $(From\ symmetry\ of\ C\ and\ definition\ of\ DC)$
$(Ext.T.3)$ $\forall x P(x, x)$ $(From\ definition\ of\ P)$
$(Ext.T.4)$ $\forall x \forall y P(x, y) \wedge P(y, x) \rightarrow EQ(x, y)$ $(From\ definition\ of\ EQ)$
$(Ext.T.5)$ $\forall x \forall y \forall z P(x, y) \wedge P(y, z) \rightarrow P(x, z)$ $(From\ definition\ of\ P)$
$(Ext.T.6)$ $\forall x EQ(x, x)$ $(From\ definitions\ of\ P\ EQ)$
$(Ext.T.7)$ $\forall x \forall y EQ(x, y) \rightarrow EQ(y, x)$ $(From\ definition\ of\ EQ)$
$(Ext.T.8)$ $\forall x \forall y \forall z EQ(x, y) \wedge EQ(y, z) \rightarrow EQ(x, z)$ $(From\ definition\ of\ EQ\ and\ Ext.T.5)$
$(Ext.T.9)$ $\forall x \forall y (x = y) \rightarrow EQ(x, y)$
$(Ext.T.10)$ $\forall x \forall y (x \neq y) \rightarrow \neg EQ(x, y)$
$(Ext.T.11)$ $\forall x \neg PP(x, x)$ $(From\ definition\ of\ PP)$
$(Ext.T.12)$ $\forall x \forall y PP(x, y) \rightarrow \neg PP(y, x)$ $(From\ definition\ of\ PP)$

$(Ext.T.13)$ $\forall x \forall y \forall z PP(x,y) \wedge PP(y,z) \to PP(x,z)$ $(From\ definition\ of\ PP\ and\ Ext.T.5)$

$(Ext.T.14)$ $\forall x O'(x,x)$ $(From\ definitions\ of\ P,\ O)$

$(Ext.T.15)$ $\forall x \forall y O'(x,y) \to O'(y,x)$ $(From\ definition\ of\ O)$

$(Ext.T.16)$ $\forall x \neg DR(x,x)$ $(From\ definitions\ of\ DC,\ O,\ DR)$

$(Ext.T.17)$ $\forall x \forall y DR(x,y) \to DR(y,x)$ $(From\ definitions\ of\ O,\ DR)$

$(Ext.T.18)$ $\forall x \neg PO'(x,x)$ $(From\ definitions\ of\ DC\ PO)$

$(Ext.T.19)$ $\forall x \forall y PO'(x,y) \to PO'(y,x)$ $(From\ definitions\ of\ PO,\ O)$

$(Ext.T.20)$ $\forall x \neg EC'(x,x)$ $(From\ definitions\ of\ DC,\ O,\ EC)$

$(Ext.T.21)$ $\forall x \forall y EC'(x,y) \to EC'(y,x)$ $(From\ symmetry\ C\ and\ definitions\ of\ O,\ EC)$

$(Ext.T.22)$ $\forall x \neg TPP(x,x)$ $(From\ definitions\ of\ DC\ TPP)$

$(Ext.T.23)$ $\forall x \forall y TPP(x,y) \to \neg TPP(y,x)$ $(From\ definitions\ of\ TPP\ and\ Ext.T.22)$

$(Ext.T.24)$ $\forall x \neg NTPP(x,x)$ $(From\ symmetry\ C\ and\ definition\ of\ NTPP)$

$(Ext.T.25)$ $\forall x \forall y NTPP(x,y) \to \neg NTPP(y,x)$ $(From\ definitions\ of\ NTPP\ Ext.T.24)$

$(Ext.T.26)$ $\forall x \forall y \forall z NTPP(x,y) \wedge NTPP(y,z) \to NTPP(x,z)$ $(From\ definitions\ of\ NTPP,\ EQ\ and\ Ext.T.5)$

$(Ext.T.27)$ $\forall x \neg CR'(x,x)$ $(From\ definitions\ of\ DC,\ O,\ CR)$

$(Ext.T.28)$ $\forall x \forall y CR'(x,y) \to CR'(y,x)$ $(From\ definition\ of\ CR)$

$(Ext.T.29)$ $\forall x \forall y B(x,y) \to TPP(x,y)$ $(From\ definition\ of\ TPP\ and\ boundary\ axiom)$

$(Ext.T.30)$ $\forall x \forall y \neg EC(x,y) \wedge \neg CR'(x,y) \leftrightarrow (C(x,y) \leftrightarrow O'(x,y))$

(From all axioms and definitions of DC, O, EC, CR)

$(Ext.T.31)$ $\forall x \forall y [\neg \exists z EC'(z,x) \wedge \neg CR'(z,x) \to [P(x,y) \leftrightarrow \forall u [O'(u,x) \to O'(u,y)]]]$

(From definitions of CR, EC, and Ext.T.5)

$(Ext.T.32)$ $\forall x \forall y \forall z NTPP(x,y) \wedge C(z,x) \to O'(z,y) \vee CR'(z,y)$

(From definitions of O, NTPP, CR, and Ext.T.5)

$(Ext.T.33)$ $\forall x \forall y \forall z P(x,y) \wedge NTPP(y,z) \to NTPP(x,z)$

(From definitions of P, NTPP, and Ext.T.5)

$(Ext.T.34)$ $\forall x \forall y C(x,y) \leftrightarrow \exists z P(z,y) \wedge C(z,x)$ $(From\ definitions\ of\ P,\ and\ Ext.T.5)$